On Spectral Efficiency of Asynchronous OFDM/FBMC based Cellular Networks

Yahia Medjahdi, Michel Terré, Didier Le Ruyet and Daniel Roviras
Electronics and Communications Laboratory, CNAM, Paris, France
{yahia.medjahdi, michel.terre, leruyet, daniel.roviras}@cnam.fr

***Abstract*—The growing demand for wireless communication makes it important to determine the capacity limits of the different multicarrier systems. In this paper, we address the impact of imperfect inter-cell synchronization on the average capacity of orthogonal frequency division multiplexing (OFDM) and filter bank based multicarrier (FBMC) multi-cellular networks. Based on computing the moment generating functions of the asynchronous interference power, a simple new expression for the exact evaluation of the average capacity is derived considering the frequency correlation fading between adjacent interfering subcarriers.**

***Index Terms*—Multi-carrier; Filter bank; asynchronous; moment generating function (MGF); correlated Rayleigh fading.**

I. Introduction

Beyond traditional voice communication, wireless networks are currently evolving to support high speed data applications such as video streaming and internet browsing [1]. However, wireless communication systems are subject to several impairments such as fading, pathloss and interference. These effects can seriously degrade the quality of service and lead to transmission failures.

An orthogonal Frequency Division Multiplexing (OFDM) system is a type of multicarrier modulation which consists of splitting up a wide band signal at a high symbol rate into several lower rate signals, each one occupying a narrower band. System performance improves because subcarriers experience flat fading channels and are orthogonal to one another thus minimizing the threat of interference. However, the OFDM performance tends to suffer from degradation because of possible episodes of imperfect time and frequency synchronization, since a loss in orthogonality can occur between subcarriers at the OFDM receiver [2, 3].

The degradation of the signal to interference plus noise ratio is a common criterion to analyze the impact of timing non-synchronization on the system performance [2, 4]. In [5], an interference modeling, based on the so called Interference Table [6], has been developed for two multicarrier techniques: CP-OFDM with a rectangular pulse shape and for Filter Bank based Multi-Carrier (FBMC) with a prototype filter designed for a better frequency selectivity using the frequency sampling technique [7].

Although interference analysis in OFDM single user has become popular in literature e.g [8, 9], the extension of this analysis to a multi-cellular environment is not so straightforward. This problem is significant for the following reasons. First, in a multi-cellular environment the interference stems from subcarriers distributed among several transmitters which require more than one random variable (RV) to model this interference, therefore, the analysis becomes more difficult. Second, in contrast to many researches based on the classical Gaussian approximation [10, 11], we cannot always rely on this approximation. Indeed, when each interferer is assigned more then one subcarrier, the resulting interference is no longer a sum of independent variables. For this reason, the Gaussian approximation becomes inaccurate [3].

Based on the interference table model introduced in [5, 6], we derive an explicit form of the average capacity of time-asynchronous OFDM and FBMC systems in the case of the block subcarrier assignment taking into account the correlation between the subchannel gains belonging to a given block subcarrier. The computation of the average capacity is based on the moment generating function of the interference power.

The rest of the paper is organized as follows. Section II is devoted to describing the system model of the downlink of OFDM and FBMC based multi-cellular networks. A brief review of interference table modeling is given in Section III. We further derive an explicit expression of the average capacity of asynchronous OFDM/FBMC systems in Section IV. Simulation results are presented and discussed in Section V. Section VI concludes the paper.

II. The System Model

We consider the downlink transmission in OFDM/FBMC based multi-cellular networks depicted in Fig. 1 (a). The reference mobile user is located at (i, j). The reference base station (BS) is assumed to be situated at the origin (i_0, j_0) =(0,0). In this analysis, we consider two tiers of the neighboring cells that are surrounding the reference mobile user. Let the k-th BS be located at (i_k, j_k), then, the distance between the reference mobile user and the k-th BS is given by

$$d_k = \sqrt{(i_k - i)^2 + (j_k - j)^2} \tag{1}$$

The cell radius is denoted by R in Fig. 1.

Concerning the frequency reuse scheme, the subcarriers are allocated according to the most common subcarrier assignment scheme, namely, the block subcarrier assignment scheme which is described in Fig. 1 (b). We assume in this scheme that δ adjacent subcarriers to each block are free and serve as guard bands between the different blocks. It should be noticed that the frequency reuse factor is 1/7.

978-1-4577-1348-4/11/$26.00 ©2011 IEEE

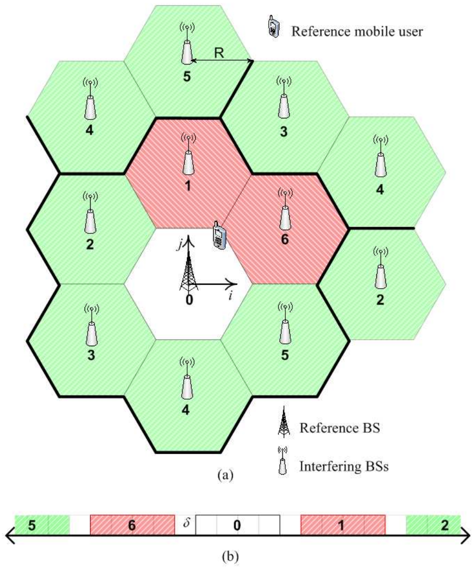

Fig. 1. (a). The downlink of OFDM/FBMC based networks (b). The subcarrier assignment scheme

The reference mobile user is assumed to be perfectly synchronized with its BS but it is not necessarily synchronized with the other BSs. We can express the composite signal at the reference receiver by the sum of the desired signal coming from the reference BS and the interference signal coming from the surrounding BSs,

$$r(t) = \underbrace{d_0^{-\beta/2} s_0(t) * h_0(t)}_{\text{desired signal}} + \underbrace{\sum_{k=1}^{K} d_k^{-\beta/2} s_k(t-\tau_k) * h_k(t)}_{\text{interference signal}} + n(t) \quad (2)$$

where

- K is the total number of neighboring cells
- $s_k(t)$ is the transmitted signal from the k-th BS
- τ_k and $h_k(t)$ denote respectively the timing offset and the impulse response of the channel between the reference mobile user and the k-th BS
- $n(t)$ is the additive white Gaussian noise (AWGN)
- β is the path loss exponent

Because of imperfect timing synchronization between the reference cell and the neighboring ones, the signals arriving from the latter will interfere with the desired signal. This interference will degrade the SINR, and this degradation will be investigated in the next section.

III. Interference and SINR Analysis

In this section we present an accurate interference analysis that considers the multipath effects on the different signals and also the timing offsets between the interfering BSs and the reference one. In contrast to direct analytical methods that require huge computational efforts. We present here an attractive non-direct analytical method that significantly reduces the complexity of the analysis.

A. Interference Tables

OFDM/FBMC interference tables are presented in Table I as a function of the subcarrier space between the interfering subcarrier and the target one (l). These tables model the correlation between subcarriers caused by the timing misalignment between the different transmitters (BSs in our analysis). It is worth noticing that this interference has been computed considering CP-OFDM system with a CP duration $\Delta = T/8$, where T is the OFDM symbol duration and FBMC system using the PHYDYAS prototype filter [7] with an overlapping factor of 4. According to Table I, we see that the interference in OFDM case is spread over a high number of subchannels; on the other hand, for the FBMC, the interference is more localized and it appears only on the subchannel of interest and the two immediate adjacent ones.

TABLE I
OFDM/FBMC Mean Interference Table[6]

$\|l\|$	OFDM	FBMC
0	**7.04E-01**	**8.24E-01**
1	**9.00E-02**	**8.78E-02**
2	**2.25E-02**	1.09E-06
3	**1.00E-03**	2.09E-08
4	**5.63E-03**	2.52E-09
5	**3.60E-03**	5.60E-10
6	**2.50E-03**	1.72E-10
7	**1.84E-03**	6.50E-11
8	**1.41E-03**	2.83E-11

B. Interference power in a selective frequency channel

It has been demonstrated in [5], that the asynchronous interference power arriving through a selective frequency channel can be calculated using the following expression

$$P_{\text{interf}}(m,\tau) = d^{-\beta} P_{trans}(m') I(\tau, |m'-m|) \left|H(m')\right|^2 \quad (3)$$

where

- d is the distance between the interferer and the victim user
- $P_{trans}(m')$ is the transmitted power on the m'-th interfering subchannel
- $I(\tau, |m'-m|)$ is the interference table coefficient corresponding to a timing offset τ and m denotes the index of the victim subchannel
- $\left|H(m')\right|^2$ is the power channel gain between the interfering transmitter and the reference receiver on the m'-th subchannel

In the multi-cell case described in Section II, the interference is caused by the K BSs surrounding the reference cell. We can easily express the total interference power occurring at the output filter of the reference mobile user by

$$P_{\text{interf}}(m, \{\tau_k, k=1,...,K\}) = \sum_{k=1}^{K} \sum_{m' \in F_k} d_k^{-\beta} P_{trans}(m') I(\tau_k, |m'-m|) \left|H_k(m')\right|^2 \quad (4)$$

where F_k denotes the set of subcarriers that are assigned to the k-th BS. We recall that τ_k and $|H_k(m')|^2$ are respectively the timing offset and the power channel gain between the reference mobile user and the k-th BS.

As aforementioned, the reference mobile user is assumed to be perfectly synchronized with its BS. Consequently, the power of the desired signal can be written as

$$P_{\text{desired}}(m) = d_0^{-\beta} P_{trans}(m) \left|H_0(m)\right|^2 \tag{5}$$

According to (4) and (5), the SINR is given by

$$\text{SINR}(m) = \frac{\left|H_0(m)\right|^2}{\sum_{k=1}^{K} \sum_{m' \in F_k} A_{k,m,m'} \left|H_k(m')\right|^2 + b} \tag{6}$$

where N_0 denotes the noise power spectral density and B_{sc} is the bandwidth of the m-th subchannel. where

$$A_{k,m,m'} = \left[\frac{d_k}{d_0}\right]^{-\beta} \frac{P_{trans}(m')}{P_{trans}(m)} I(\tau_k, |m' - m|)$$

$$b = \frac{N_0 B_{sc}}{d_0^{-\beta} P_{trans}(m)} \tag{7}$$

It should be noticed that we consider the transmitted power $P_{trans}(m)$, of each BS, as a constant. The coefficient $A_{k,m,m'}$ can thus be written as follows,

$$A_{k,m,m'} = \left[\frac{d_k}{d_0}\right]^{-\beta} I(\tau_k, |m' - m|) \tag{8}$$

IV. Capacity Analysis

In this section, we pay our attention to derive a closed-form expression of the capacity considering the asynchronous interference caused by the surrounding BSs. In the literature, the capacity is given by Shannon's well-known formula [12] when the decision variables are Gaussian random variables,

$$C(\text{SNR}) = B \log_2 (1 + \text{SNR}) \tag{9}$$

where B denotes the channel bandwidth.
Therefore, by conditioning on the set of variables $\{H_0(m), H_k(m'), \forall k, m, m'\}$ and substituting (6) in (9), we can obtain the exact closed form for the conditional capacity in the presence of interference (10).

$$C(\text{SNR})|_{H_0(m),H_k(m')} = B \log_2 \left(\frac{\left|H_0(m)\right|^2}{\sum_{k=1}^{K} \sum_{m' \in F_k} A_{k,m,m'} \left|H_k(m')\right|^2 + b} \right) \tag{10}$$

In order to reduce the complexity of computing the average capacity which requires $K \times N$ integrations into only one integration, we refer to the following lemma [13], which is based on the moment generating function of the interference power.

Lemma[13]: Let x be a unit-mean gamma random variable (RV) with parameter α, and let g be an arbitrary non-negative random variable that is independent of x. Then

$$E_{x,g}\left[\ln\left(\frac{x}{g+b}\right)\right] = \int_0^{+\infty} \left[\frac{1}{z} - \frac{1}{z(1+z)^{\alpha}}\right] \mathcal{M}_g(\alpha z) e^{-z\alpha b} dz \tag{11}$$

where $\mathcal{M}_g(z) = E_g\left[e^{-zg}\right]$ is the moment generating function (MGF) of g.
As $|H_0(m)|$ is a Rayleigh random variable, $x = \left|H_0(m)\right|^2$ is an exponential RV with a probability density function (pdf) $f(x) = e^{-x}$. In other words, x is a unit-mean gamma RV with $\alpha = 1$. Therefore, the expression (11) becomes

$$E_{x,g}\left[\ln\left(\frac{x}{g+b}\right)\right] = \int_0^{+\infty} \frac{1}{(1+z)} \mathcal{M}_g(z) e^{-zb} dz \tag{12}$$

In our analysis, the random variable related to the total interference power is defined by

$$g = \sum_{k=1}^{K} \sum_{m' \in F_k} A_{k,m,m'} g_{k,m'} \tag{13}$$

where $g_{k,m'} = \left|H_k(m')\right|^2$.
As the signals coming from the different interfering cells $\{\sum_{m' \in F_k} A_{k,m,m'} g_{k,m'}, \forall k\}$ are independent, the moment generating function of g is given by

$$\mathcal{M}_g(z) = \prod_{k=1}^{K} \mathcal{M}_k(z) \tag{14}$$

where

$$\mathcal{M}_k(z) = E_{\{g_{k,m'}, m' \in F_k\}}\left[e^{-z \sum_{m' \in F_k} A_{k,m,m'} g_{k,m'}}\right] \tag{15}$$

However, the RVs $\{g_{k,m'} = \left|H_k(m')\right|^2, m' \in F_k\}$ are correlated because they belong to the same cluster used by the k-th BS. To deal with this problem, let Ω_k defined by

$$\Omega_k = \left[\rho_{i,j}\right]_{(i,j) \in F_k \times F_k}$$

where $\rho_{i,j} = \rho_{j,i}$, be the square root of the variance-covariance matrix of the RVs $\{g_{k,m'}, m' \in F_k\}$.

Following [14], the MGF $\mathcal{M}_k(z)$ is obtained by

$$\mathcal{M}_k(z) = \left|I_{L_k} + \Omega_k D_k^A z\right|^{-1} \tag{16}$$

where I_{L_k} is the $L_k \times L_k$ identity matrix and L_k denotes the cardinal of F_k. D_k^A is a diagonal matrix of diagonal elements,

$$D_k^A(i,i) = A_{k,m,i} \qquad i \in F_k \tag{17}$$

Substituting (16) in the expression (14), we obtain the MGF related to the total interference RV g defined in (13),

$$\mathcal{M}_g(z) = \prod_{k=1}^{K} \mathcal{M}_k(z) = \prod_{k=1}^{K} \left|I_{L_k} + \Omega_k D_k^A z\right|^{-1} \tag{18}$$

Therefore, using the expressions (12) and (18), the final expression of the average capacity for K interfering BSs is shown in (19).

$$C_{\text{average}} = \frac{B}{\ln(2)} \int_{0}^{+\infty} \frac{e^{-zb}}{1+z} \prod_{k=1}^{K} \left| I_{L_k} + \Omega_k \, D_k^A z \right|^{-1} dz \quad (19)$$

V. Simulation Results

In the previous section, we have derived closed-form expressions of the average capacity in the downlink of an asynchronous K-cell network. In contrast to direct complex analytical methods, these expressions present an efficient approach to compute the average capacity with a significantly reduced computational complexity. In this section, we present numerical results for the downlink of OFDM and FBMC systems with the block subcarrier scheme as described in Section II. The reference mobile user is located at the top of the cell as shown in Fig. 1. The cell radius in our simulation is R= 1km.

We have considered the Pedestrian-A model as a Rayleigh fading propagation channel [15]. The choice of this model is based on the assumption that the subcarriers of interest experience flat fading channels. Therefore, we can focus on the impact of the asynchronous inter-cell interference because the intra-cell interference in the FBMC case is negligible. The path loss of a received signal at a distance d is governed by the following expression [16] corresponding to a path loss exponent $\beta = 3.76$ and a carrier frequency of 2 GHz

$$\Gamma_{loss}(d) = 128.1 + 37.6 \log_{10}(d)[\text{dB}]$$

On the other hand, we consider a system with $N = 1024$ subcarriers using a total bandwidth of 10 MHz. The noise term is characterized by a thermal noise density of -174 dBm/Hz. The prefix cyclic duration is fixed at $\Delta = T/8$, and the size of the subcarrier block is set at 18 subcarriers. For the FBMC system, we recall that we use the PHYDYAS prototype filter with an overlapping factor of 4 [7]. It is worth mentioning that the following results are compared to the perfect synchronized (PS) scenario in which the orthogonality between the different subchannels is maintained.

In Fig. 2, we investigate the accuracy of the capacity expression (19). The average capacity of OFDM and FBMC modulations are plotted against the SNR, in absence of a guard band between the clusters of the different cells ($\delta = 0$). Both theoretical and simulation results are displayed in Fig. 2. The theoretical results are evaluated using (19). The exact theoretical results depicted in Fig. 2 show an excellent match to the corresponding simulation results. In this case, we assume that the timing offset τ is a uniform RV in the interval [0,T]. Fig. 2 also shows that the timing synchronization errors cause a severe degradation in the average capacity. Moreover, this degradation becomes large when increasing the SNR level. We can also see a capacity floor at high SNR values for asynchronous OFDM systems. This observation can be explained by the fact that the noise level is negligible compared to the asynchronous interference caused by the other BSs.

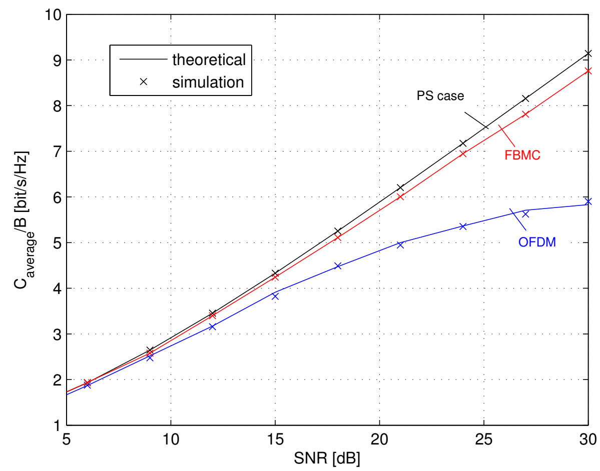

Fig. 2. The OFDM/FBMC average capacity against the SNR for $\tau \in$[0,T], the guard-band size δ =0

Such a case is expected in the interference-limited scenarios. On the other hand, we observe a better performance of the asynchronous FBMC when compared to the asynchronous OFDM. Such a gain can be explained by the fact that only the two subcarriers on the edge suffer from the interference caused by their adjacent subcarriers in the FBMC case. However, in the OFDM case, the entire cluster suffers from the interference caused by all neighboring clusters. In the perfect synchronized case, both modulation schemes lead to identical results which means that the actual bit rate is higher for FBMC because it does not use CP.

In Fig. 3, we plot the average capacity versus the SNR with different timing offset scenarios: the perfect synchronized case in scenario (a), [0,T/4] in scenario (b) and [0,T] in scenario (c).

In the OFDM system, the degradation is severe and increases when the timing error interval is larger. We can explain this result as follows: when the timing offset is lower than the cyclic prefix duration $\tau \in$[0,Δ], the orthogonality between the different clusters is maintained; otherwise the reference user will suffer from an asynchronous interference. Since the timing offset is a uniform random variable, the probability obtaining the performance of the perfect synchronized case is given by the CP duration over the whole timing offset interval (Δ/τ_{max}). The probability of the orthogonality decreases as τ_{max} increases. Therefore, the average capacity becomes lower. On the other hand, the FBMC system is not sensitive to the timing offset interval length. Such a result can be explained by the fact that the interference at the two subcarriers of the edge is roughly invariable with respect to the timing offset value. It should be noticed that Fig. 3 also shows an excellent match between the simulation and theoretical results obtained by the closed-form expression of the average capacity given in (19).

prototype filter.

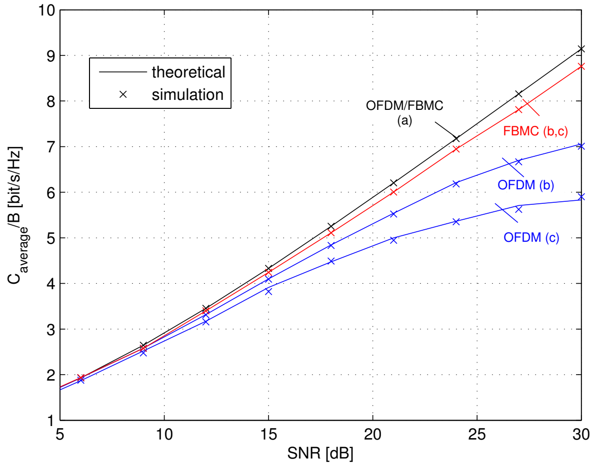

Fig. 3. The average capacity for different timing offset intervals (a). PS (b). $\tau \in$[0,T/4] (c). $\tau \in$[0,T], the guard-band size δ =0

VI. Conclusion

In this paper, we have investigated the impact of timing synchronization errors on the performance of the downlink of OFDM and FBMC based multi-cellular networks. We first give a brief review of the interference table model. We then develop a theoretical derivation of the average capacity expression. In contrast to the direct analytical method that requires huge computational efforts, the obtained closed-form expression reduces significantly the computation complexity. The accuracy of the obtained expression has been validated through the different simulation results. A global evaluation has been performed taking into account the timing error range parameter. Through this evaluation, we have shown that in OFDM case, timing errors between BSs cause a severe degradation in system performance. This result is explained by the loss of orthogonality between all system subcarriers. In contrast to the OFDM system, the FBMC waveform is demonstrated to be less sensitive to timing errors between the different cells, due to the better frequency localization of the prototype filter.

References

[1] T. Zahariadis and D. Kazakos. "(R)evolution towards 4G mobile communication systems," *IEEE Wireless Comm. Mag.*, 10 (4):6-7, August 2003.

[2] Y. Mostofi and D. Cox, "Mathematical Analysis of the Impact of Timing Synchronization Errors on the Performance of an OFDM System," *IEEE Trans. Commun.*, vol. 54, no. 2, Feb. 2006.

[3] K. A. Hamdi and Y. Shobowale, "Interference Analysis in Downlink OFDM Considering Imperfect Inter-cell Synchronization," *IEEE Trans. Veh. Technol.*, vol. 58, no. 7, pp. 3283-3291, Sep. 2009.

[4] R. Raghunath and A. Chockalingam, "SIR analysis and interference cancellation in uplink OFDMA with large carrier frequency and timing offsets," *in IEEE Wireless Commun. Netw. Conf.*, Kowloon, China, Mar. 2007, pp. 9961001.

[5] Y. Medjahdi, M. Terré, D. Le Ruyet, D. Roviras, "Asynchronous OFDM/FBMC Interference Analysis in Selective Channels", *in Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, Sep. 2010, pp. 538-542.

[6] Y. Medjahdi, M. Terré, D. Le Ruyet, D. Roviras, J.A. Nossek and L. Baltar, "Inter-Cell Interference Analysis for OFDM/FBMC Systems", *Proc. of IEEE-SPAWC'09 conference*, Jun. 2009, pp. 598-602.

[7] M. G. Bellanger, "Specification and design of a prototype filter for filter bank based multicarrier transmission," in *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, May 2001, pp. 2417-2420.

[8] T.Pollet, M.Van Bladel and M. Moeneclaey "BER Sensitivity of OFDM Systems to Carrier Frequency Offset and Wiener Phase Noise," *IEEE Trans. Commun.*, vol. 43, no. 2/3/4, pp.191-193, Feb-Apr. 1995.

[9] T.Pollet and M. Moeneclaey "The Effect of Carrier Frequency Offset on the Performance of Band limited Single Carrier and OFDM Signals," in *Proc. IEEE GLOBECOM*, Nov. 1996, pp 719-723.

[10] X. Wang, T. T. Tjhung, Y. Wu, and B. Caron, "SER performance evaluation and optimization of OFDM system with residual frequency and timing offsets from imperfect synchronization," *IEEE Trans. Broadcast.*, vol. 49, no. 2, pp. 170-177, Jun. 2003.

[11] J. Cheng and N. C. Beaulieu, "Accurate DS-CDMA bit-error probability calculation in Rayleigh fading," *IEEE Trans. Wireless Commun.*, vol. 1, no. 1, pp. 3-15, Jan. 2002.

[12] C. E. Shannon, "A Mathematical Theory of Communication," *Bell. Sys. Tech. Journal*, pp. 379-423, 1948.

[13] K. A. Hamdi, "A Useful Technique for Interference Analysis in Nakagami Fading", *IEEE Trans. Commun.*, vol 55, no. 6, June 2007.

[14] A.S. Krishnamoorthy and M. Parthasarathy, "A multi-variate gamma-type distribution", *Annals of Mathematical Statistics*, pp. 549-557, 1951.

[15] Rec. ITU-R M.1225 1, *Guidelines for evaluation of radio transmission technologies for IMT-2000*, 1997.

[16] ETSI TR 125 942, *Universal Mobile Telecommunications System (UMTS); Radio Frequency (RF) system scenarios*, 2010.