Optimizing the deployment of a multilevel optical FTTH network

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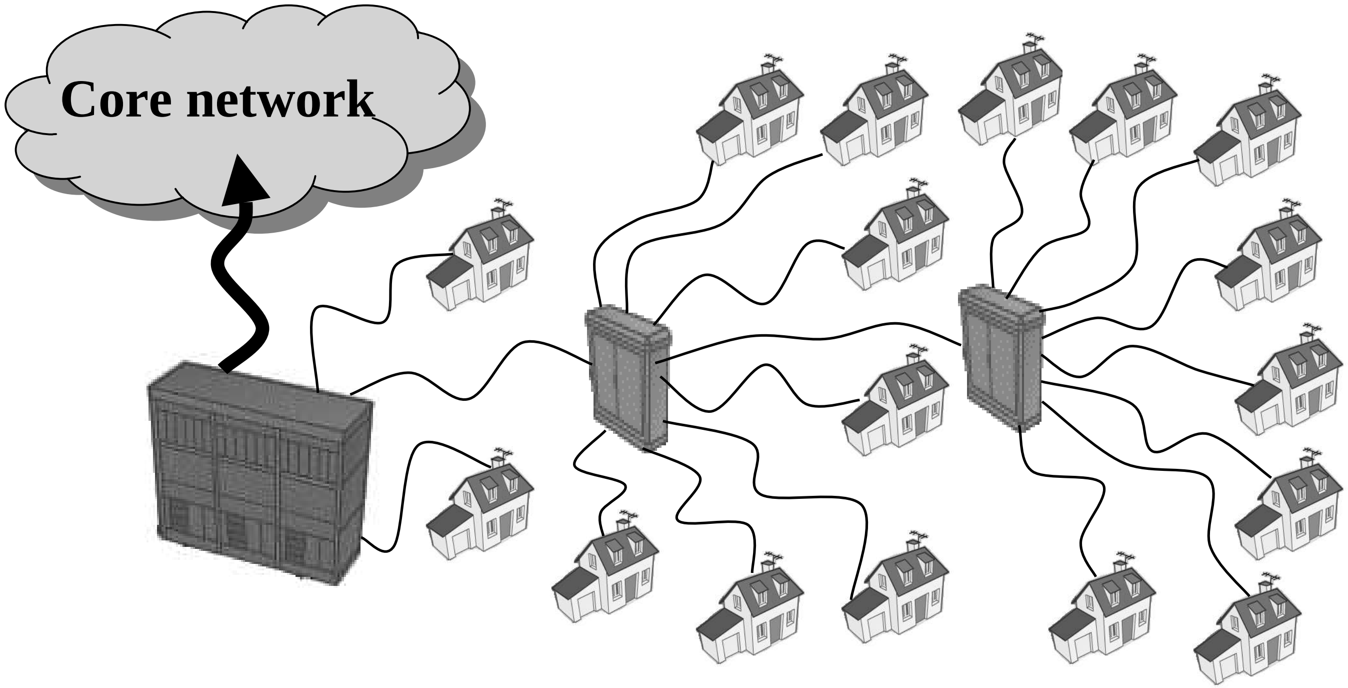
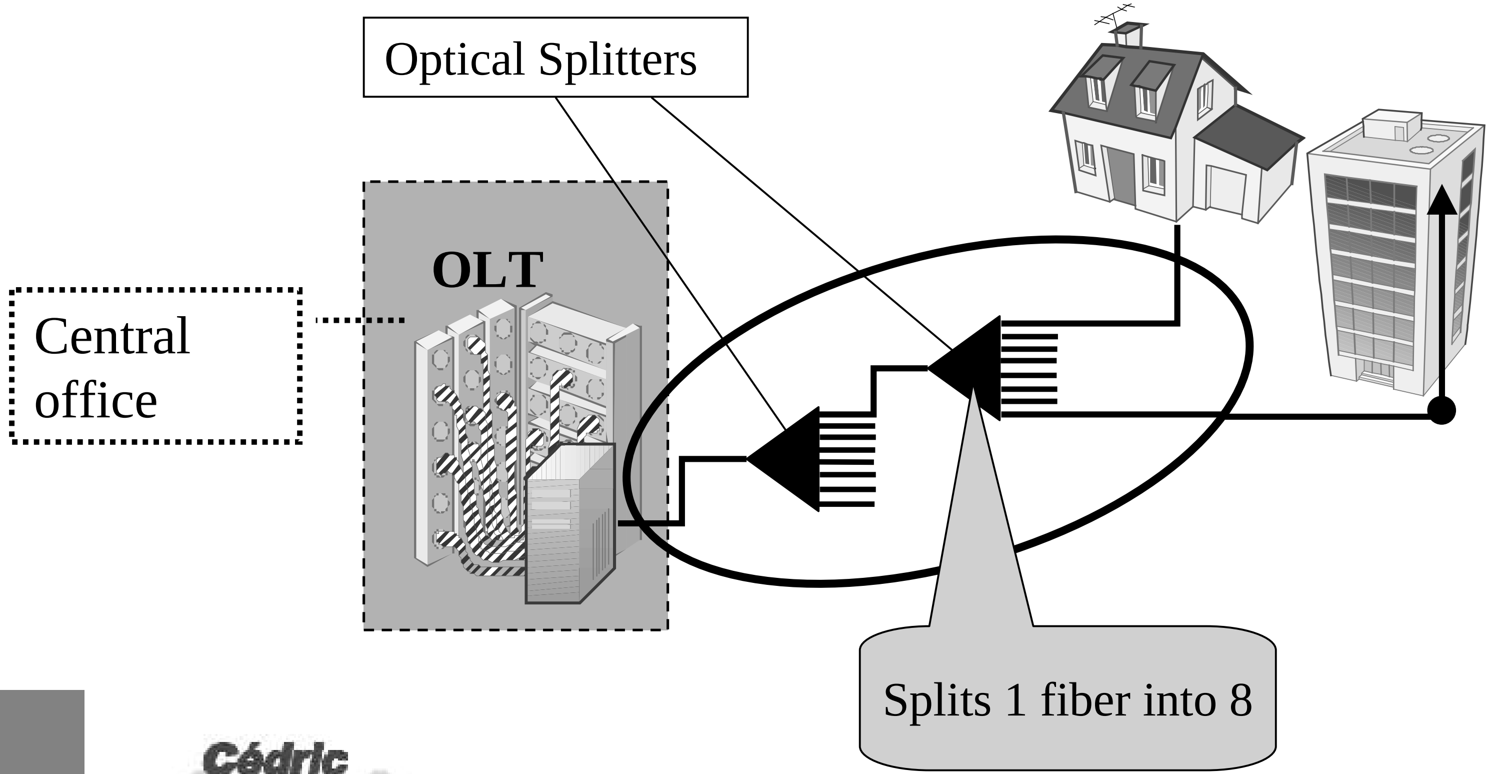
Context

- IP telecommunication networks
- Access Network: hierarchical network that links clients to the network
- Equipments: central office, optical splitters, optical fibers



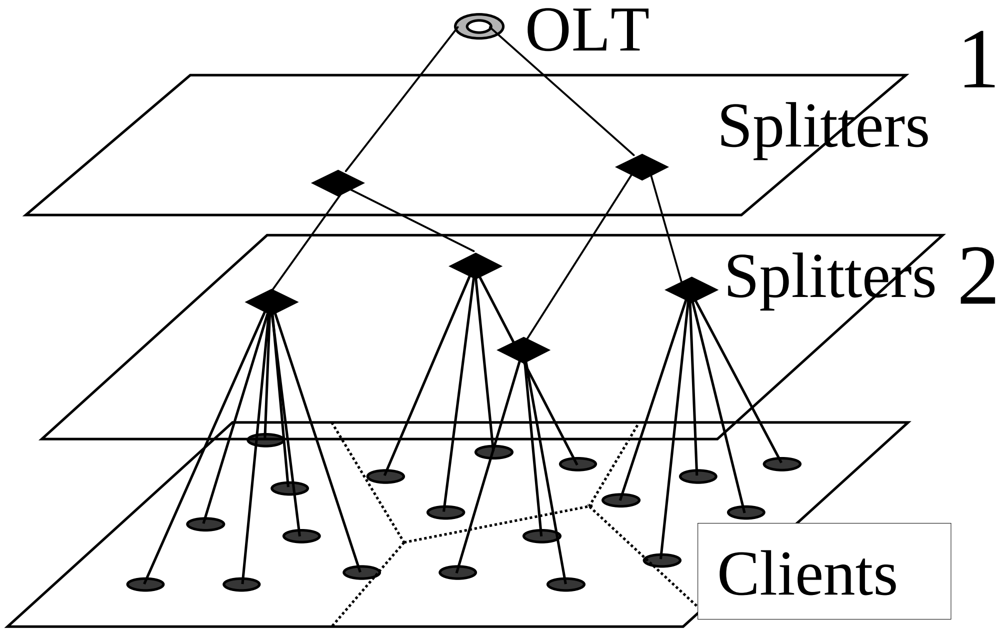
Architecture

- Architecture GPON (Gigabit Passive Optical Network)
  - Optical Line Termination connects the clients
  - two levels of optical splitters distribute fibers to the clients



Hierarchical network

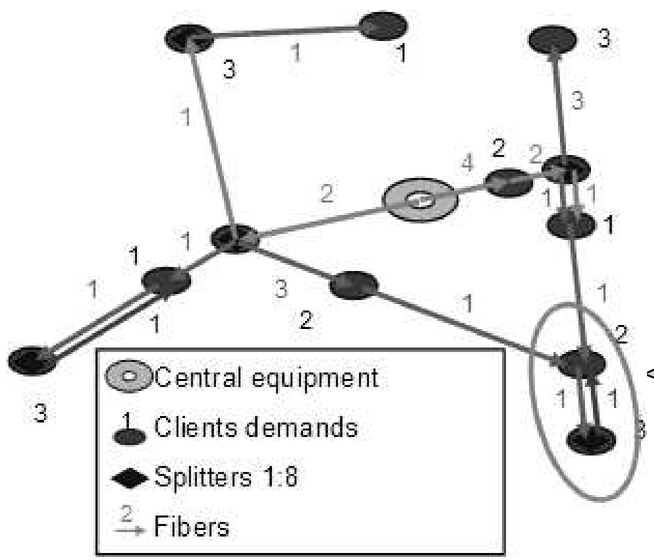
- OLT = central office
- splitters = intermediate passive equipments on 2 levels



Tree structure for the splitters

Infrastructure graph

- locate the equipments in a graph that modelizes ducts in a city
  - locate splitters at the nodes (two levels of splitter)
  - route fibers linking OLT, splitters, clients



Fibers may induce cycles

The datas

- A graph modelizing a local zone
- A capacity on each edge (maximum number of fibers routed on the edge)
- the client demands at the nodes
- the « node 0 » location of the central office

The datas

C^k cost of a level k splitter

l_{ij} length of the edge $[i, j]$

γ^k linear cost of a level k fiber

d_{ij}^k cost of a level k fiber on the edge $[i, j]$, $d_{ij}^k = l_{ij}\gamma^k$

m^k number of fibers produced by a level k splitter

a_i demand in fibers of the clients at node i

b_{ij} capacity of the edge $[i, j]$

The decision variables of the problem

z_i^k number of level k splitters installed at node i

f_{ij}^k number of level k fibers routed on edge $[i, j]$

u_i^k number of unused fibers of level k at node i

The mathematical model

Integer linear problem

$$\min_{f,z,u} \sum_{i=0}^{n} \sum_{k=1}^{2} C^k z_i^k + \sum_{[i,j] \in E} \sum_{k=1}^{3} d_{ij}^k \left(f_{ij}^k + f_{ji}^k \right)$$

s.t.

$$\begin{cases} \sum_{j/[j,i] \in E} f_{ji}^1 = z_i^1 + \sum_{j/[i,j] \in E} f_{ij}^1 & i = 1,\dots,n \quad (1) \\ \sum_{j/[j,i] \in E} f_{ji}^2 + m^1 z_i^1 = z_i^2 + \sum_{j/[i,j] \in E} f_{ij}^2 + u_i^2 & i = 0,\dots,n \quad (2) \\ \sum_{j/[j,i] \in E} f_{ji}^3 + m^2 z_i^2 = a_i + \sum_{j/[i,j] \in E} f_{ij}^3 + u_i^3 & i = 0,\dots,n \quad (3) \\ \sum_{k=1}^{3} \left(f_{ij}^k + f_{ji}^k \right) \leq b_{ij} & [i,j] \in E \quad (4) \\ z_i^k, u_i^k, f_{ij}^k \text{ integer} & \end{cases}$$

Valid inequalities

Q set of points $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $\begin{cases} \beta x + y \geq \alpha \\ x, y \text{ integer} \end{cases}$ with β , α integer

Integer division of α by β : $\alpha = q \times \beta + r$ with $0 \leq r < \beta$

Then $rx + y \geq r(q+1)$ is valid for Q

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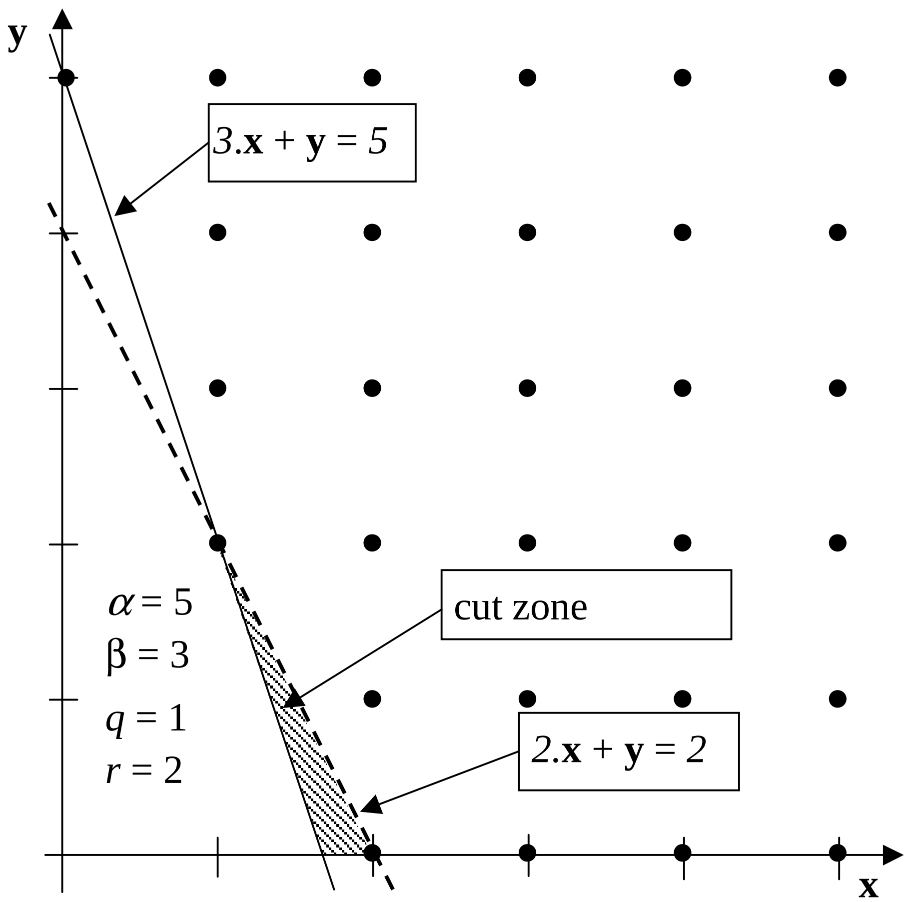
Cutting plane

$$\begin{cases} \beta x + y \geq \alpha \\ x, y \text{ integer} \end{cases}$$

$$\alpha = q \times \beta + r$$

$$rx + y \geq r(q + 1)$$



How to use these cuts in our problem?

We consider a set of nodes A and we add equations (3) for $i \in A$

$$\sum_{j/[j,i]\in E} f_{ji}^3 + m^2 z_i^2 = a_i + \sum_{j/[i,j]\in E} f_{ij}^3 + u_i^3 \quad i \in A$$

$$\sum_{i\in A}\left(\sum_{j\notin A/[j,i]\in E} f_{ji}^3 + m^2 z_i^2\right) = \sum_{i\in A}\left(a_i + \sum_{j\notin A/[i,j]\in E} f_{ij}^3 + u_i^3\right)$$

$$\sum_{i\in A}\left(\sum_{j\notin A/[j,i]\in E} f_{ji}^3\right) + m^2 \sum_{i\in A} z_i^2 \geq \sum_{i\in A} a_i$$

Aggregate some inequalities

We put

$$x = \sum_{i\in A} z_i^2 \quad , \quad y = \sum_{i\in A}\left(\sum_{j\notin A/[j,i]\in E} f_{ji}^3\right) \quad , \quad \beta = m^2 \quad , \quad \alpha = \sum_{i\in A} a_i$$

With

$$\beta x + y \geq \alpha$$

We obtain a valid inequality

$$rx + y \geq r(q+1)$$

Then use the cutting inequality

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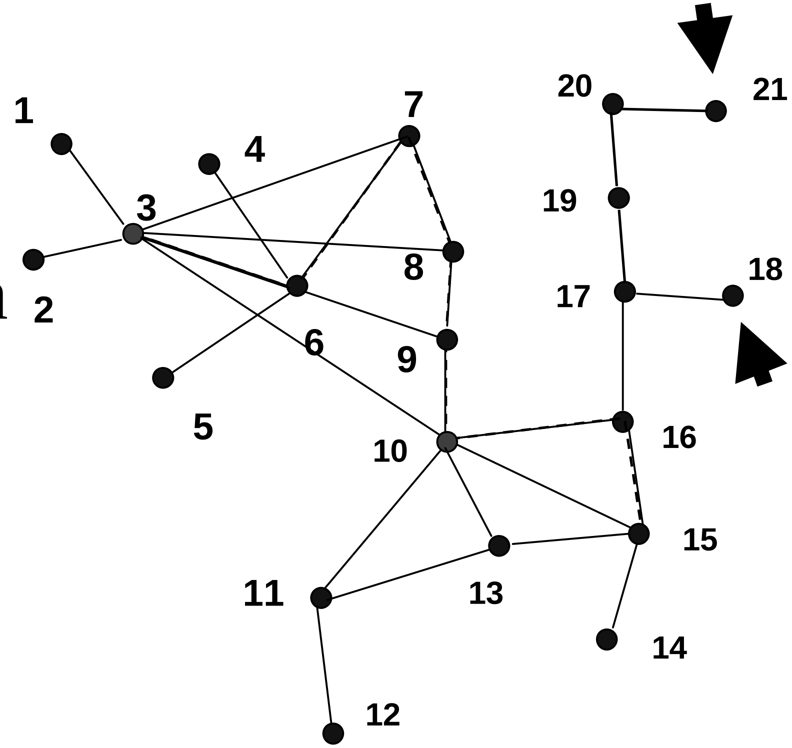
Graph reduction

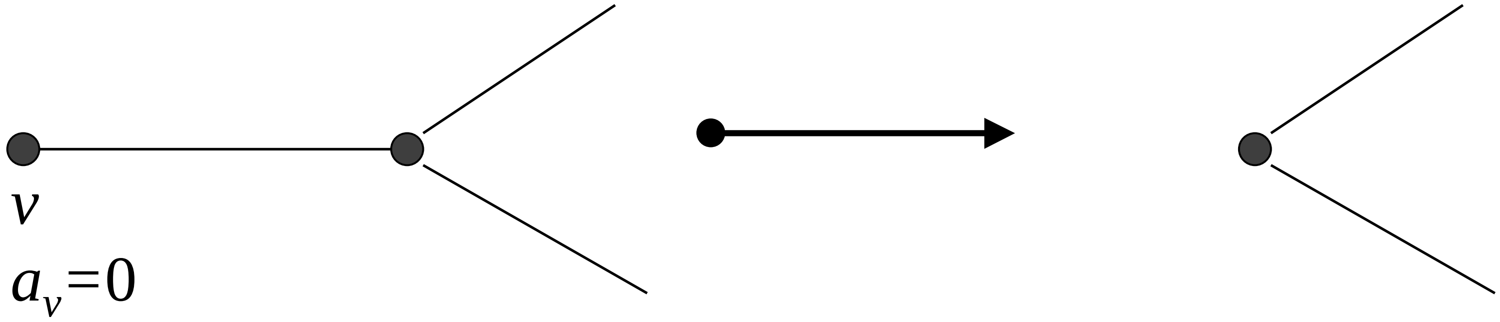
- Many nodes of the infrastructure graph are only geographic node with no client
  - some of them are not useful for optimizing the deployment
  - they can be removed from the graph



Graph reduction

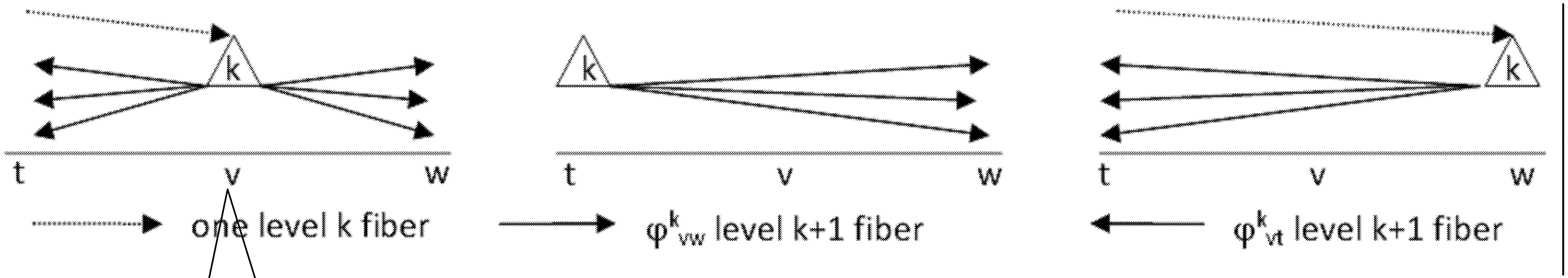
Trivial reduction

A leaf of the graph (node of degree 1) with no demand can be removed of the graph



Graph reduction

Nodes of degree 2



Node *v* with 2 neighbors

Graph reduction

Theorem

If the linear costs of fibers of two levels k and k' satisfy $\gamma^k \leq 2\gamma^{k'}$

then there is an optimal solution with no splitter on any node v of degree 2

with $a_v = 0$ (no demand)

Numerical tests

Real instances

Instance	Existing infrastructure $\|V\|$	$\|E\|$	Fiber demand nb_{Client}	d_{Client}
Data_1	342	375	184	72.7
Data_2	920	1000	570	73.0
Data_3	1072	1163	667	78.2
Data_4	932	951	583	75.9
Data_5	1478	1614	1010	77.2
Data_6	712	772	441	82.6
Data_7	3044	3337	2061	79.0
Data_8	1265	1365	497	26.2
Data_9	2853	3139	1301	25.2
Data_10	844	905	327	21.1
Data_11	2076	2280	973	24.7
Data_12	901	996	347	24.1
Data_13	181	218	46	31.7
Data_14	3276	3639	1652	25.9

Numerical tests

Synthesis of the results (after one hour)

Instance	Size (B&B tree)	UB	LB	Gap (%)
Data_1	616009	80598	80528	0.08
Data_2	876711	257089	256324	0.30
Data_3	954926	302175	301203	0.32
Data_4	702446	265894	265249	0.24
Data_5	598893	452732	451166	0.35
Data_6	11729991	199857	199434	0.21
Data_7	283240	931339.6	922673	0.93
Data_8	538494	158896	156673	1.40
Data_9	209571	383986	377728	1.63
Data_10	662166	88318	87050	1.44
Data_11	291528	281501	276454	1.79
Data_12	633018	106634	105340	1.20
Data_13	1039895	21163	20902	0.57
Data_14	138503	504803	496829	1.58

Data_1 – Data_7: average gap = 0.35%

Data_8 – Data_14: average gap = 1.37%

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Numerical tests

Impact of graph reduction schemes

Instance	Preprocessed graph $\|V^{red}\|$	$\|E^{red}\|$	nb_{Client}	Reduction indicators $nb_{removed}$	red_V (%)	red_E (%)
Data_1	103	136	57	239	70	64
Data_2	260	340	178	660	72	66
Data_3	288	379	195	784	73	67
Data_4	278	368	176	654	70	64
Data_5	393	529	291	1085	73	67
Data_6	200	260	129	512	72	66
Data_7	808	1101	586	2236	73	67
Data_8	533	633	285	732	58	54
Data_9	1253	1539	719	1600	56	51
Data_10	365	426	200	479	57	53
Data_11	897	1101	534	1179	57	52
Data_12	408	503	215	493	55	49
Data_13	107	144	33	74	41	34
Data_14	1624	1987	922	2125	57	52

average reductions 63% 58%

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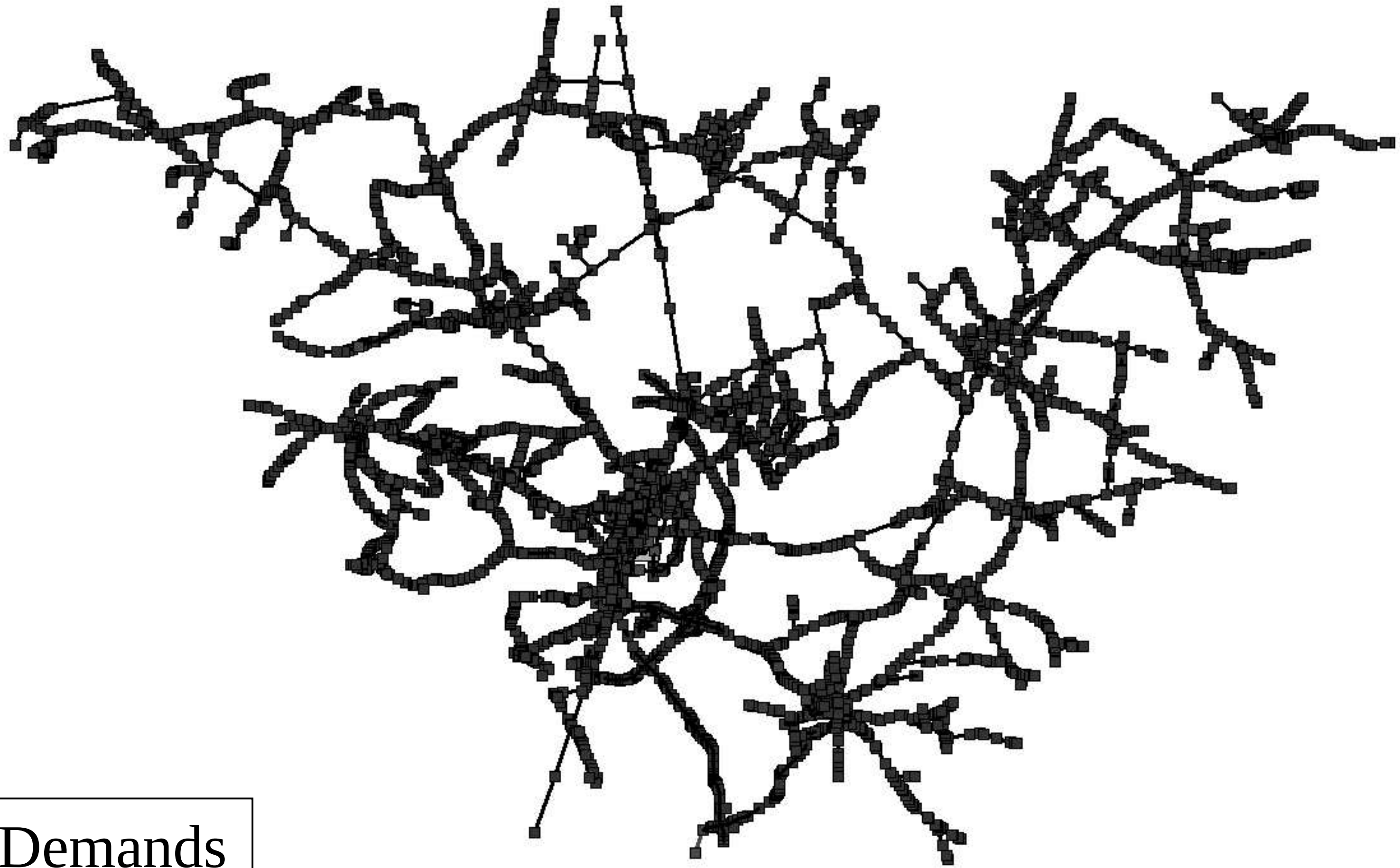
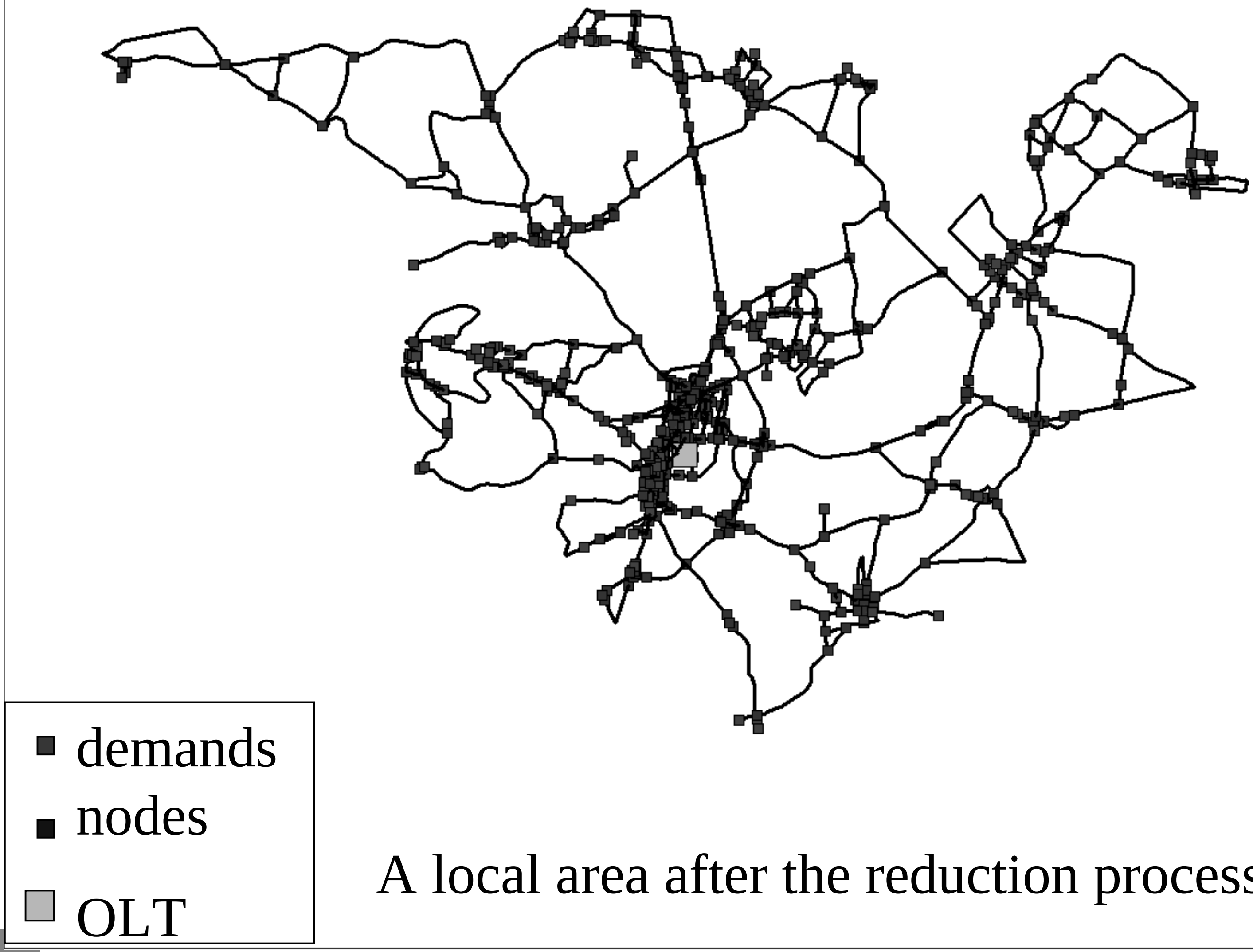


- Demands
- nodes
- OLT

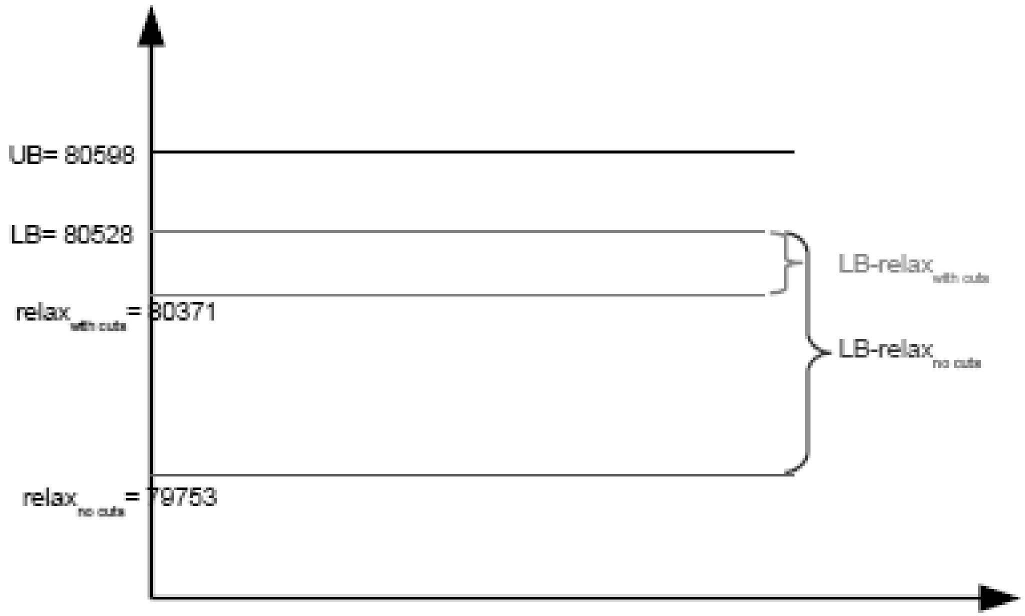
A local area before applying the reduction scheme



A local area after the reduction process

Numerical tests

Do the valid inequalities reduce the gap obtained without cut?



$gap_{LB} = (relax_{with\ cuts} - relax_{no\ cut}) / (LB - relax_{no\ cut})$

Numerical tests

Impact of valid inequalities

Instance	UB	LB	Continuous relaxation			% Gap closed	
			$\text{relax}_{no\ cuts}$	$\text{relax}_{with\ cuts}$	gap (%)	gap_{UB}	gap_{LB}
Data_1	80598	80528	79753	80371	0.8	73.1	79.7
Data_2	257089	256324	254210	256231	0.8	70.2	95.6
Data_3	302175	301203	298904	301094	0.7	67.0	95.3
Data_4	265894	265249	262716	265142	0.9	76.3	95.8
Data_5	452732	451166	446894	450731	0.9	65.7	89.8
Data_6	199857	199434	197653	199321	0.8	75.7	93.7
Data_7	931339	922673	914984	922567	0.8	46.4	98.6
Data_8	158896	156673	151955	156481	3.0	65.2	95.9
Data_9	383986	377728	370449	377614	1.9	52.9	98.4
Data_10	88318	87050	84433	86821	2.8	61.5	91.2
Data_11	281501	276454	271180	276352	1.9	50.1	98.1
Data_12	106634	105340	102805	105226	2.4	63.2	95.5
Data_13	21163	20902	20587	20902	1.5	54.7	100.0
Data_14	504803	496829	486279	496694	2.1	56.2	98.7

average closed gap 57.7% 96.8%

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Conclusion

- Access Network, GPON architecture
- Design of an integer linear program
- We solve real instances from medium size to large size

- In future work, taking into account uncertainty of the demand

Thank you.