# Optimal Multi-Crop Planning Implemented Under Deficit Irrigation

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***Abstract*—Multi-Crop planning (MCP) optimization model for cropping pattern and water allocation is introduced as a nonlinear programming problem. Its solution promotes an efficient use of water with a flexibility to keep the chosen crops at either full or deficit irrigation throughout different stages so that the net financial return is maximized within certain production bounds and resources constraints. The problem-solution approach is as follows: at first a preliminary mathematical tools are presented involving existence, benchmark linear models and a relaxation formulation, second two meta-heuristic algorithms Simulated Annealing (SA) and Particle Swarm Optimization (PSO) are implemented as a numerical technique for solving the MCP problem. The particularity of our approach consists of using the solution of the linear problem as an initial guess for the SA, while for PSO the particle swarm is initiated in the neighberhood of that solution.**

***Keywords*—Multi-crop planning, deficit irrigation, nonlinear programming, simulated annealing, particle swarm optimization***

## I. Introduction

THE demand on water is increasing exponentially due to extraordinary population and industrial growth. the supply is, therefore, far less than the actual demand and further its existence is being threatened by the adverse effects of climate change. Water resources management in the next decade is inevitable and should be every nation's primary objective. In fact, there is a growing interest to develop advanced management methods to prevent wasting water in the course of satisfying human needs, protecting health, and ensuring food production, restoring of ecosystems, as well as for social economic evolvement and for sustainable development. Programmes are launched by the European Union through the Common Agricultural Policy (CAP) or internationally through the Consortium of International Agricultural Research Centers (CGIAR) in order to overcome these crucial problems.

Nowadays, there is a great urge to new irrigation technologies in agricultural research. So multiple optimization methods are suggested to: find the right crop selection, implement crop rotation and schedule precise irrigation. Traditionally, agricultural models primarily focused on maximizing the yield and the economic return per unit area by allocating water to different crops according to their water needs [1, 2, 3]. With time, studies have switched to deficit irrigation and its impact on crop yield production. The objective was regulating deficit irrigation in a way to save water by subjecting crops to periods of moisture stress with minimal effects on yields. Within this approach and based on FAO report [4], it is seen that, the reduction in the yield may be little, compared with the benefits obtained through diverting the saved water to cover wider cropped area. The study in [5] claims that optimal irrigation is useful in increasing the crop production, the irrigated area and the net economic returns.

In this paper, a Decision- Support Tool (DST) based on a Non Linear Programming (NLP) model for optimal multi-crop planning is proposed . The aim is to maximize the net financial returns. In fact, the authors presented a mathematical programming model with an objective function inspired by relations found in [6] and [7], and taking into account water limitation at each time period as a constraint as in [5]. Furthermore, another restriction ought to be considered is the crop production quota which is important to preserve crop diversity (Greening rule - CAP). Otherwise, farmers will grow the most profitable plant leading to agricultural surplus in some crops and shortage in others. Nevertheless, this greening rule will ensure market stability and will secure availability of supplies. In response to the above conditions, the Multi-Crop Planning (MCP) model is presented and utilized to find an optimal water allocation and crop pattern to maximize profit.

In the past decade, comprehensive studies have been conducted on Evolutionary Algorithms (EA) for solving nonlinear programming problems concerning optimal crop planning and irrigation water allocation. Genetic Algorithm (GA) [8, 9] was used to solve the irrigation problem, while [10] search for the optimal irrigation reservoir operation using Simulated Annealing (SA). On the other hand [11] and [12] applied Particle Swarm Optimization (PSO) algorithm to find the optimal reservoir operation for the irrigation of multiple crops. However, the problem-solution approach presented in this investigation is as follows: 1- Preliminary mathematical tools are addressed that involves solution existence, linear models extraction and a relaxation formulation of (NLP), afterward 2- Two meta-heuristic algorithms SA and PSO are implemented as a numerical technique for solving the MCP problem. The approach trait is using the solution of the linear problem as an initial guess for the SA, while for PSO the particle swarm is initiated in the neighborhood of that solution, rather than generating them randomly as in [10] and [11]. The effectiveness of the suggested models is tested and evaluated using proposed data.

The remaining of the article is organized as follows: Section II describes in details the MCP problem while section III presents the suggested technique used in solving it. A numerical example and some results are discussed in section IV, whereas the drawn conclusion of the study and the future work are presented in section V.

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## II. Multi-Crop Planning Model Description

The MCP is designed to obtain the optimal farm plan for allocating irrigation areas in a multi-cropping system. The objective is to maximize net profit of the produced yield over the planning horizon when conflicts emerge between the supplied amount of water and the actual demand during the irrigation season. This problem holds within, decision variables involving the choice of crops, their acreage and their water allocation at every time interval. A feasible solution must satisfy a multiple constraints regarding land area, irrigation and crop production .

TABLE I. Notations and indices for (MCP) problems

| Notation | Definition | Unit |
|---|---|---|
| (MCP) model | | |
| $l$ | Number of stages | Unitless |
| $n$ | Number of crops | Unitless |
| $X_i$ | Area of the land planted with crop $i$ | $ha$ |
| $P_i$ | Profit obtained when crop $i$ is assigned to area $X_i$ | euro |
| $ETa_{ij}$ | Actual evapotranspiration of crop $i$ at stage $j$ | $mm$ |
| $Etm_{ij}$ | Potential evapotranspiration of crop $i$ at stage $j$ | $mm$ |
| $WA_{ij}$ | Applied water amount to crop $i$ at stage $j$ | $m^3/ha$ |
| $WR_{ij}$ | Required water amount for crop $i$ at stage $j$ | $m^3/ha$ |
| $r_j$ | Available amount of water at stage $j$ | $m^3$ |
| $Ya_i$ | Produced amount of crop $i$ | $Kg/ha$ |
| $Ym_i$ | Potential amount of crop $i$ | $Kg/ha$ |
| $ky_{ij}$ | Yield response factor | Unitless |
| $\lambda_{ij}$ | Index of sensitivity of crop $i$ at stage $j$ | Unitless |
| $c_i$ | Fixed amount of crop $i$ can't exceed | $Kg$ |
| $p_i$ | Selling price of crop $i$ | euro/ $Kg$ |
| $B_i$ | Cost of used water by crop $i$ | euro$/ha$ |
| $pw$ | Cost of water | euro$/m^3$ |
| $C_i$ | Misc. costs to plant crop $i$ | euro$/ha$ |
| $A_{total}$ | Total area of agricultural activity | $ha$ |

### A. Objective Function

The objective function of the MCP model is the net profit from crops production that is calculated by subtracting the total cost (manual labor, seeds, fertilizers, water used...etc.) from the market value of the yield. However, as a first step, one should establish the water-crop relationship which contains timing, quantity of water applied and the effects of crop-water stress for deficit irrigation at different growth stages. A widely used relation was presented by Jensen [6] and it is expressed in the following formula:

$$\frac{Ya_i}{Ym_i} = \prod_{j=1}^{l} \left( \frac{ETa_{ij}}{ETm_{ij}} \right)^{\lambda_{ij}} \tag{1}$$

Whereas [13] presents a linear relationship between relative yield and relative evapotranspiration. It empirically derives yield response factors ($k_y$) for individual growth stages (i.e. establishment, vegetative, flowering, yield formation and ripening). However, Jensen's model (1) can be applied at time steps smaller than the growth stages. Its sensitivity indices are related to the yield response factors ($k_y$) that represents the effect of a reduction in evapotranspiration on yield losses, by the following polynomial [10]:

$$\lambda_{ij} = 0.2418(ky_{ij})^3 - 0.1768(ky_{ij})^2 + 0.9464(ky_{ij}) - 0.0177$$

The polynomial is obtained for a coefficient of determination $R^2 = 0.999$.

In our work, we didn't consider the water balance in soil. According to [7], it can be assumed that the ratio of the actual crop evapotranspiration to potential crop evapotranspiration is the same as the ratio of irrigation supply to demand, that is:

$$\frac{ETa_{ij}}{ETm_{ij}} = \frac{WA_{ij}}{WR_{ij}} \tag{2}$$

Then combining equations (1) and (2), we get:

$$Ya_i = Ym_i \prod_{j=1}^{l} \left( \frac{WA_{ij}}{WR_{ij}} \right)^{\lambda_{ij}} \tag{3}$$

Thus the profit obtained from planting crop $i$ can be determined using the formula:

$$P_i = \left[ p_i Ya_i - (B_i + C_i) \right] X_i \tag{4}$$

Writing $P_i$ in terms of the variables $X_i$ and $WA_{ij}$, we get:

$$P_i = \left[ p_i Ym_i \prod_{j=1}^{l} \left( \tfrac{WA_{ij}}{WR_{ij}} \right)^{\lambda_{ij}} - (pw \sum_{j=1}^{l} WA_{ij} + C_i) \right] X_i \tag{5}$$

Hence the objective function is given by:

$$F = \sum_{i=1}^{n} P_i \tag{6}$$

### B. Constraints

The objective function is considered to be bounded by a set of constraints, regarding water limitations at each stage, total land area and crop production quota:.

1) If the amount of water available at each time step or stage is limited to a fixed quantity $r_j$, for $j = 1, ..., l$, then it is important to consider water limit constraint:

$$\sum_{i=1}^{n} X_i \cdot WA_{ij} \leq r_j, \text{ for all } j = 1, ..., l \tag{7}$$

However, in a region with abundant water resources, the availability of water is not a problem. In this case, water is provided with no limits, so this constraint is no more restrictive and could be omitted.

2) Under deficit irrigation, the applied water can not exceed the required amount,

$$WA_{ij} \leq WR_{ij}, \text{ for all } i = 1, ..., n \;\&\; j = 1, ..., l \tag{8}$$

whereas, in state of full irrigation equality is assumed.

3) The produced yield of crop $i$ can not be greater than a fixed quantity $c_i$,

$$X_i Ya_i \leq c_i, \text{ for all } i = 1, ..., n \tag{9}$$

This condition is important for two main reasons: it maintains crops diversity and keeps the market values of crops stable. In fact, any over production of certain crop can cause a decrease in price which is here not the case.

4) Area constraint

$$\sum_{i=1}^{n} X_i \leq A_{total} \tag{10}$$

5) Non-negativity constraints

$$WA_{ij} \geq 0, \text{ for } i = 1, ..., n \text{ \& } j = 1, ..., l \tag{11}$$

$$X_i \geq 0, \text{ for } i = 1, ..., n \tag{12}$$

### C. The Nonlinear Optimization Problem

In the foregoing, the decision variables are the planted areas $X_i's$ and the applied water at each stage $WA_{ij}$. Let us denoted by $S \subset \mathbb{R}^{n+l\times n}$ the set of all points that satisfy constraints (7-12), then the Nonlinear Programming (NLP) problem becomes:

$$\max_{x \in S} F(x)$$

## III. Suggested Solution Approach

### A. Mathematical Tools

In the following and before proceeding to the numerical part, a preliminary theoretical study was carried out. It involves existence, benchmark linear models and relaxation formulation of the main (NLP) problem.

*Proposition 1:* (Existence) The (NLP) problem admits a solution in $S$.

*Proof:* Since $\lambda_{ij} > 0$, the function $F$ defined from $\mathbb{R}^{n+l\times n}$ into $\mathbb{R}$ is a continuous function. Moreover it is obvious that the set $S$ is closed and bounded, thus $S$ is a compact set. By Weierstrass theorem, $F$ attains its global maximum in $S$. ■

*Proposition 2:* In case of no water limits and full irrigation the (NLP) problem is reduced to a Linear Programming problem denoted by (LP1) and has the form:

$$\text{(LP1)} \begin{cases} \max\limits_{X} f^T X \\ \sum_{i=1}^{n} X_i \leq A_{total} \\ 0 \leq X_i \leq c_i / Ym_i, i = 1, ..., n \end{cases}$$

where $f$ is a vector in $\mathbb{R}^n$ such that for every $i = 1, ..., n$ ; $f_i = p_i Ym_i - (pw \sum\limits_{j=1}^{l} WR_{ij} + C_i)$, $X = (X_1, X_2, ..., X_n)$.

*Proof:* No water limits with full irrigation means that, constraint (7) is omitted and we have $WA_{ij} = WR_{ij}$ for all $i = 1, ..., n$ and $j = 1, ..., l$. So, $\frac{WA_{ij}}{WR_{ij}} = 1$, $Ya_i = Ym_i$. In this case the profit from crop $i$ becomes $f_i = p_i Ym_i - (pw \sum\limits_{j=1}^{l} WR_{ij} + C_i)$. As a result the objective function and the nonlinear constraints become linear, and the problem is transformed into the form:

$$\begin{cases} \max\limits_{X} \sum\limits_{i=1}^{n} f_i X_i \\ X_i Ym_i \leq c_i, \forall i = 1, ..., n \\ \sum\limits_{i=1}^{n} X_i \leq A_{total} \\ X_i \geq 0, \forall i = 1, ..., n \end{cases}$$

By setting $f = (f_1, f_2, ..., f_n)$, $X = (X_1, X_2, ..., X_n)$. The above linear problem is reshaped into (LP1). ■

*Proposition 3:* Suppose full irrigation is considered within a limited amount of available water. (NLP) is transformed into a Linear programming model (LP2) that has the form:

$$\begin{cases} \max\limits_{X} f^T X \\ \sum\limits_{i=1}^{n} X_i \cdot WR_{ij} \leq r_j, \ \forall \ \ j = 1, ..., l \\ \sum\limits_{i=1}^{n} X_i \leq A_{total} \\ 0 \leq X_i \leq c_i / Ym_i, \forall i = 1, ..., n \end{cases}$$

*Proof:* The proof is carried in same way as that of proposition 2, but constraint (7) should be put back due limited amount of water. ■

The solution of the (LP1) and that of (LP2) model will present the optimal crop distribution among the areas in two different scenarios: the first one, full irrigation with no water limits while the second case is, full irrigation with water amount limitations.

*Remark 1:* These two models are intended to provide a reference for the (NLP) problem. In fact, (LP2) gives a lower bound for the (NLP) objective function, that is:

$$\max_{x \in S} F(x) \geq \max_{X \in S_2} f^T X$$

where $S_2$ is the set of all points that satisfy constraints of (LP2).

*Recall 1:* Denote by $E$ the objective function to be maximized over a domain $D$. A relaxation of a maximization problem,

$$z = \max \{E(x);\ x \in D \subset \mathbb{R}^n\}$$

is another maximization problem of the form,

$$z_R = \max \{E_R(x);\ x \in D_R \subset \mathbb{R}^n\}$$

with the following properties: $D_R \supseteq D$ and $E_R(x) \geq E(x)$ for all $x \in D$.

Let us rearrange $F$ :

$$F = \sum_{i=1}^{n} \left[ p_i Ya_i - (pw \sum_{j=1}^{l} WA_{ij} + C_i) \right] X_i$$

$$F = \sum_{i=1}^{n} p_i Y a_i X_i - \sum_{i=1}^{n} (pw \sum_{j=1}^{l} WA_{ij} + C_i) X_i$$

Now define the new objective function:

$$F_R = \sum_{i=1}^{n} p_i \min(Y a_i X_i, c_i) - \sum_{i=1}^{n} (pw \sum_{j=1}^{l} WA_{ij} + C_i) X_i \tag{13}$$

*Proposition 4:* The nonlinear programming problem (RNLP) defined as:

$$\max_{x \in S_R} F_R(x)$$

is the relaxed version of the (NLP) problem. where $S_R \subset \mathbb{R}^{n+l\times n}$ is the set of points that statisfy constraints (7-8, 10-12).

*Proof:* It is clear that $S \subset S_R$ and for all $x \in S$, we have $F_R(x) = F(x)$. Thus (RNLP) is the relaxation of (NLP) problem. ■

### B. Models solution

The optimization of the objective functions addressed in both (NLP) and (RNLP) is a problem without obvious analytical solution and perhaps with multiple local optimum. However, in the recent years Evolutionary Algorithms (EA) have become popular tools for nonlinear optimization problems. In fact two Evolutionary Algorithms are used in this work: the first is the Particle Swarm Optimization (PSO) and the second is the Simulated Annealing (SA) algorithm.

*1) Particle Swarm Optimization:* Particle Swarm Optimization was first presented by Kennedy and Eberhart in 1995. The PSO search procedures are based on the swarm concept (inspired by social behaviors of bird flocking or fish schooling), which is a group of individuals that are able to optimize certain fitness function. Every individual can transmit information to another and ultimately allow the entire group to move towards the same object or in the same direction. It is a method to simulate the behavior of individuals of the species who work for the benefit of the whole group.

PSO is initialized with a population of random solutions creating a particle swarm and searches for optima by updating generations. Each particle keeps track of its coordinates in the search space which are related to the best solution (fitness) it has reached so far. This value is referred to $Pbest$. Another "best" value that is tracked by the particle swarm optimizer is the best value, obtained so far in all of the particle swarm. This best value is a global best and is named $Gbest$ .To find the optimal solution, each particle moves in the direction of its $Pbest$ and $Gbest$. After continuous iterations, the particle swarm will gravitate towards the optimum solution. The parts of PSO are given below:

(1) *Velocity update*

$$v_i^{k+1} = \omega v_i^k + c_1 \cdot \mathrm{rand} \cdot (Pbest_i^k - x_i^k) + c_2 \cdot \mathrm{rand} \cdot (Gbest^k - x_i^k)$$

- $c_1$ and $c_2$ are learning factors of PSO
- $\mathrm{rand}$ is a random number uniformly distributed
- $Pbest_i^k$ individual best optima for particle $i$ after $k$ iterations
- $Gbest^k$ group optima after k iterations
- $\omega$ inertia weight
- $v_i^{k+1}$ velocity of particle $i$ in iteration $k+1$
- $x_i^k$ position of particle i in iteration $k$

(2) *Position update*

- $$x_i^{k+1} = x_i^k + v_i^{k+1}$$

(3) *Weighting*

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega \min}{Iter_{\max}} \cdot Iter$$

- $\omega_{\max}$ largest weight
- $\omega_{\min}$ smallest weight
- $Iter$ iterative times
- $Iter_{\max}$ maximum iterative times for PSO

*2) Simulated Annealing:* The basic concept of SA was inspired from statistical thermodynamics by American physicist Metropolis in 1953. Whereas Kirkpatrick in 1983 suggested using this concept for finding solutions for optimization problems, and was the first literature to successfully utilize SA in combinatorial optimized problems. the SA is theoretically guaranteed to converge to the global optimal solution under certain assumptions and given infinite execution time. In practice, however, globally optimal or near-optimal solutions can be obtained in a large yet finite number of iterations [10].

A general description of the algorithm is given below. Let us denote by $E(x)$ the function being minimized, where $x$ is the vector of decision variables, of dimension $d$. The basic steps of the algorithm are the following:

1. Choose the initial temperature $T_0$ and an initial state $x_0$, calculate its energy $E_0$; set the step number $k = 0$
2. Find a feasible candidate state $x_{k+1} = x_k + \triangle x$, $\triangle x$ is randomly generated from a normal distribution of mean 0 and variance $T_k$
3. Calculate $E_{k+1}$ and the energy difference $\triangle E = E_{k+1} - E_k$
4. If $\triangle E < 0$, state improvement, update solution; otherwise accept it, if $\mathrm{random(0,1)} \leq \exp(-\triangle E/T_k)$
5. Lower the temperature with the geometric cooling scheme proposed by Kirkpatrick et al, $T_{k+1} = \alpha T_k$, $\alpha \in (0, 1)$
6. Increment k and repeat steps 2–6 until $k$ satisfies some specified stopping criterion.

An essential parameter that should be considered during the implementation of SA is the initial temperature $T_0$. If it is set too low, the randomly generated candidate states will never be far from the initial state and the search space will not be properly explored. This may lead to convergence to a local minimum. On the other hand, if it is set too high, the vast majority of candidate states will be rejected as infeasible and the generation procedure will be very inefficient. Kirkpatrick suggested that a suitable initial temperature is one that results in an average increase acceptance probability of about 0.8. The value of $T_0$ will clearly depend on the scaling of cost function and hence the determination of $T_0$ is problem-specific.

### C. Implementation and Approach

The solution approach is carried through the following steps:

- Solve (LP2). Denote by $X^* = (X_1^*, ..., X_n^*)$ solution of (LP2) and by $WX^* = vec(\chi_i \cdot WR_{ij})$, where $\chi_i = \begin{cases} 0 & \text{if} \quad X_i^* = 0 \\ 1 & \text{if} \quad X_i^* \neq 0 \end{cases}$ and vec of a matrix is a linear map. which converts the matrix into a column vector
- Solve (NLP) and (RNLP) using PSO by initiating a particle swarm in the neighborhood of $(X^*, WX^*)$.
- Resolve (NLP) and (RNLP) using SA with $(X^*, WX^*)$ as initial guess.
- Compare the different approaches then choose the best.

## IV. Simulations and Discussions

### A. Numerical Example

In this part, we shall perform an experimental evaluation for each algorithm to compute the optimal cropping pattern and irrigation scheduling for a lot of six crops. However, optimal solution determination requires knowledge about the area of agricultural region $A_{total}$ , availability of water $(r_j)$ at each stage $j = 1, 2, 3, 4$ during the irrigation season and of course the crop characteristics $(k_y)$.

Under the given conditions concerning the available water resources and crop production, the models were tested over a spread acreage covering 322 acres with available total amount of water over the four stages equal to 245000 $m^3$. The programs were coded in MATLAB language and ran on Intel Core i7-5500U CPU @ 2.40 GHZ, 12.0 GB RAM. Further, the main parameters needed for the optimization procedure and other critical data for the (NLP) and (RNLP) models are obtained after some experimentation. For example the initial temperature parameter for the SA was set to 23 based on Kirkpatrick suggestion . This is assumed to be high enough to avoid getting stuck to local maxima and to allow the initial exploration of the solution space without generating excessive numbers of infeasible candidate states. The initial guess for SA algorithm is either 1- randomly generated or 2- the solution of the (LP2) problem. For the PSO algorithm, the number of particles was set to 1000 where the particles position were initialized using either one of the two approaches, 1- randomly over the whole search space or 2- with a uniformly distributed random vector in the neighborhood of the solution obtained by the (LP2) model. Each scenario, for the nonlinear models, is run for 5 times. The top results are presented in Table II.

### B. Results and Discussions

In Table II, the recommended optimum crop pattern for each model aside with the consumed water amount are presented. Well, in the best case, the net profit gained is equal to €626240 , when full irrigation is applied in a state of water abundance. Whereas, implementing the same strategy in case of limited water resources, the optimal net profit has decreased to €345430 and the crop spread area has reduced from 100% to 44.3%. However, considering the water limitation (245000 $m^3$), the authors of this paper turned to deficit irrigation and crop pattern re-arrangement. Despite the water shortage, which is 41.5% less than the water consumed (418600 $m^3$) in the best case, the optimal net financial returns obtained by the nonlinear programming models has reached 65.2% with respect to the profit obtained by (LP1) (Figure 1). Regarding the crop spread area, it has improved to cover 65.4 % of the total area. Moreover, it is found that crops 4 and 5 have the highest plantation areas according to solutions of (LP1) and (LP2). Therefore, it leads us to consider them as dominating crops. Motivated by these dominating crops, the SA and PSO algorithms was launched with the aid of the solution of (LP2) as a starting point for the schemes NLP_SA1, RNLP_SA1 and NLP_PSO1. In fact, this guarantees that the achieved profit in worst case scenario is at least that of (LP2) and not below.

Since the water supplied to the farmers is less than desired. The solution that could be adopted by a farmer is the one that provides the greatest income. In this case and based on the obtained results, the RNLP_SA1 scheme is recommended.

In what follows, the performance measure for both algorithms SA and PSO is examined for different initialization methods, using the Coefficient of Variation (CV). We recall, CV is defined as:

$$CV = \frac{\sigma}{\mu}$$

where $\mu$ is the mean and $\sigma$ standard deviation. In fact, CV exhibits the extent of variability in relation to the mean.

Table III shows, after running each scheme 5 times, the mean of the maximized objective functions and the CV along with the average execution time. It is obvious that when the algorithm was initialized with the aid of the solution of (LP2) problem, the maximized objective functions exhibit smaller deviation from the mean. However, when the algorithm was initialized with a random starting point, it was less reliable and in most of the times it had converged to a local maximum.

According to Tables II and III, the result of RNLP_SA1 came very promising regarding profit maximization and solution variability when compared with the rest of approaches for the nonlinear models. The RNLP_SA1 has possessed the highest profit with the lowest variance at a reasonable execution time. However, if one is seeking a faster running time in the nonlinear problem category, the NLP_PSO1 may be the next preferred choice. The execution time, on average, has been reduced by 50.9% relative to NLP_PSO and by 62% relative to RNLP_SA1. In contrast, the top maximized profit obtained by NLP_PSO1 was €401373 with a mean value of €367387 and $CV = 0.075$. This small average profit in comparison with the other ones may be explained by the fact that, basic PSO can be easily trapped into a local optimum. Another competing alternative regarding runtime is (LP2). Its solution is obtained almost instantly and the profit achieved is just 10% behind that of RNLP_SA1 (Figure 1). Well, (LP2) gives us a chance to gain a closer insight about the dominating crops especially in case when decisions are to be made in a very short time period.

In a word, RNLP_SA1 is an excellent tool for solving the MCP problem, it achieves high accuracy with moderate execution time. However, according to [14] increasing the number of variables the SA will converge rather slowly in order to provide sufficient moves carried out in every variable

TABLE II. EXPERIMENTAL RESULTS

| **Scenerio** Name | **Problem** | **Algorithm Used** | **Initialization** Starting point | **Area in Acres of each crop i** 1 | 2 | 3 | 4 | 5 | 6 | **Irrigated Area (ac)** | **Consumed Water** ($m^3$) | **Profit** (€) | **Elapsed Time (sec)** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LP1 | LP1 | Interior-Point | Random | 0 | 0 | 0 | 166.7 | 114.0 | 41.0 | 322 | 418600 | 626240 | 0.02 |
| LP2 | LP2 | Interior-Point | Random | 0 | 0 | 0 | 28.6 | 114.0 | 0 | 142.6 | 185710 | 345430 | 0.03 |
| NLP_SA | NLP | Simulated Annealing | Random | 12.7 | 0.4 | 0.8 | 83.4 | 108.5 | 10.1 | 215.9 | 242700 | 400297 | 817 |
| RNLP_SA | RNLP | Simulated Annealing | Random | 51.5 | 0.5 | 0.5 | 77.4 | 117.1 | 1.2 | 248.2 | 241740 | 402555 | 2052 |
| NLP_PSO | NLP | Particle Swarm Optimization | Random | 20.8 | 2.6 | 5.2 | 125.3 | 91.4 | 46.9 | 292.2 | 245000 | 346512 | 854 |
| NLP_SA1 | NLP | Simulated Annealing | Sol. LP2 | 9.2 | 1.3 | 3.1 | 82.3 | 121.2 | 0.2 | 217.4 | 241420 | 405356 | 759 |
| RNLP_SA1 | RNLP | Simulated Annealing | Sol. LP2 | 6.1 | 1.2 | 0.5 | 85.1 | 115.2 | 2.5 | 210.5 | 241500 | 408199 | 1008 |
| NLP_PSO1 | NLP | Particle Swarm Optimization | Sol. LP2 | 3.6 | 1.3 | 1.3 | 70.9 | 116.4 | 1.8 | 195.3 | 232230 | 401373 | 498 |

direction. So NLP_PSO1 may provide an extra tool, but with trade off between accuracy and runtime.

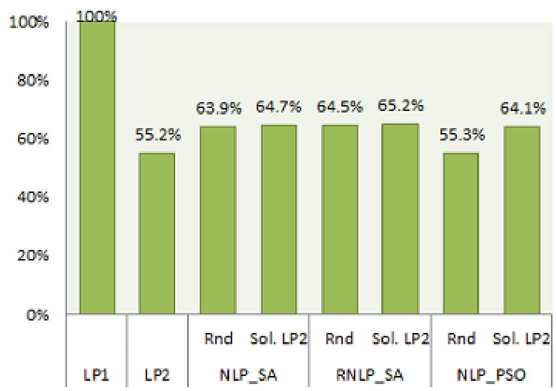


Fig. 1. Profit function comparison between different models

TABLE III. PERFORMANCE OF EACH SCHEME

| **Model i** | Mean Objective Function | Coefficent of Variation | Mean Elapsed Time | Coefficent of Variation |
|---|---|---|---|---|
| NLP_SA | 383206 | 0.048 | 461 | 0.556 |
| RNLP_SA | 367594 | 0.139 | 1985 | 0.778 |
| NLP_PSO | 305785 | 0.119 | 983 | 0.187 |
| NLP_SA1 | 396460 | 0.016 | 561 | 0.493 |
| RNLP_SA1 | 404858 | 0.009 | 1269 | 0.388 |
| NLP_PSO1 | 367388 | 0.075 | 483 | 0.020 |

## V. CONCLUSION

This work presents some mathematical tools aside with a numeric approach for resolving the Multi Cropping Planning (MCP) problem. Firstly, we establish a nonlinear programming (NLP) model that describes the MCP problem. Then two linear formulations and a relaxed version were extracted from the (NLP) model. Based on the provided numerical example, results obtained by simulated annealing and particle swarm optimization algorithms that were initiated near the solution of a specific linear problem revealed, for certain schemes, a significant decrease in algorithms execution time and an increase in the cropped area and total farming financial income under deficit irrigation. The computational results lead us to consider, in the future work, the real capabilities of the suggested approach. It will be illustrated through its implementation with real data obtained from the Bekaa-Valley region near Qaraoun reservoir- Lebanon, whereas the availability of water at each stage is tightly linked to hydropower production.

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