Representing Imprecise Time Intervals in OWL 2

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Abstract. *Representing and reasoning on imprecise temporal information is a common requirement in the field of Semantic Web. Many works exist to represent and reason on precise temporal information in OWL; however, to the best of our knowledge, none of these works is devoted to imprecise temporal time intervals. To address this problem, we propose two approaches: a crisp-based approach and a fuzzy-based approach. (1) The first approach uses only crisp standards and tools and is modeled in OWL 2. We extend the 4D-fluents model, with new crisp components, to represent imprecise time intervals and qualitative crisp interval relations. Then, we extend the Allen's interval algebra to compare imprecise time intervals in a crisp way and inferences are done via a set of SWRL rules. (2) The second approach is based on fuzzy sets theory and fuzzy tools and is modeled in Fuzzy-OWL 2. The 4D-fluents approach is extended, with new fuzzy components, in order to represent imprecise time intervals and qualitative fuzzy interval relations. The Allen's interval algebra is extended in order to compare imprecise time intervals in a fuzzy gradual personalized way. Inferences are done via a set of Mamdani IF-THEN rules.*

Keywords. Imprecise time interval • Linked data • OWL • Allen's interval algebra • 4D-fluents

1 Introduction

In the Semantic Web field, representing and reasoning on imprecise temporal information is a common requirement. Indeed, temporal information given by users is often imprecise. For instance, if they give the information "Alexandre was married to Nicole by 1981 to late 90" two measures of imprecision are involved. On the one hand, the information "by 1981" is imprecise in the sense that it could mean approximately from 1980 to 1982; on the other hand, the information "late 90" is imprecise in the sense that it could mean, with an increasingly possibility, from 1995 to 2000. When an event is characterized by a gradual beginning and/or ending, it is usual to represent the corresponding time span as an imprecise time interval.

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In OWL, many works have been proposed to represent and reason on precise temporal information; however, to the best of our knowledge, there is no work devoted to represent and reason on imprecise temporal time intervals. In this paper, we answer this problem with two approaches, one using a crisp environment, the other one using a fuzzy environment.

The first approach involves only crisp standards and tools. To represent imprecise time intervals in OWL 2, we extend the so called 4D-fluents model (Welty and Fikes 2006) which is a formalism to model crisp quantitative temporal information and the evolution of temporal concepts in OWL. This model is extended in two ways: (1) It is enhanced with new crisp components for modeling imprecise time intervals. (2) It is enhanced with qualitative temporal expressions representing crisp relations between imprecise temporal intervals. To reason on imprecise time intervals, we extend the Allen's interval algebra (Allen 1983) which is the most used and known formalisms for reasoning about crisp time intervals. We generalize Allen's relationships to handle imprecise time intervals with a crisp view. The resulting crisp temporal interval relations are inferred from the introduced imprecise time intervals using a set of SWRL rules (Horrocks et al. 2004), in OWL 2.

The second approach is based on fuzzy sets theory and fuzzy tools. It is based on Fuzzy-OWL 2 (Bobillo and Straccia 2011) which is an extension of OWL 2 that deals with fuzzy information. To represent imprecise time intervals in Fuzzy-OWL 2, we extend the 4D-fluents model in two ways: (1) It is enhanced with new fuzzy components to be able to model imprecise time intervals. (2) It is enhanced with qualitative temporal expressions representing fuzzy relations between imprecise temporal intervals. To reason on imprecise time intervals, we extend Allen's work to compare imprecise time intervals in a fuzzy gradual personalized way. Our Allen's extension introduces gradual fuzzy interval relations e.g., "long before". It is personalized in the sense that it is not limited to a given number of interval relations. It is possible to determinate the level of precision that should be in a given context. For instance, the classic Allen relation "before" may be generalized in *N* interval relations, where "before(1)" means "just before" and gradually the time gap between the two imprecise intervals increases until "before(N)" which means "long before". The resulting fuzzy interval relations are inferred from the introduced imprecise time intervals using the FuzzyDL reasoner (Bobillo and Straccia 2008), via a set of Mamdani IF-THEN rules, in Fuzzy-OWL 2.

The current paper is organized as follows: Section 2 is devoted to present some preliminary concepts and related work in the field of temporal information representation in OWL and reasoning on time intervals. In Section 3, we introduce our crisp-based approach for representing and reasoning on imprecise time intervals. In Section 4, we introduce our fuzzy-based approach for representing and reasoning on imprecise time intervals. Section 5 draws conclusions and future research directions.

2 Preliminaries and Related Work

In this section, we introduce some preliminary concepts and related work in the field of temporal information representation in OWL and reasoning on time intervals.

2.1 Representing Temporal Information in OWL

Five main approaches are proposed to represent time information in OWL: Temporal Description Logics (Artale and Franconi 2000), Versioning (Klein and Fensel 2001), N-ary relations (Noy and Rector 2006) and 4D-fluents (Welty and Fikes 2006). All these approaches represent only crisp temporal information in OWL. Temporal Description Logics extend the standard description logics with additional temporal constructs e.g., "sometime in the future". N-ary relations approach represents an N-ary relation using an additional object. The N-ary relation is represented as two properties each related with the new object. The two objects are related to each other with an N-ary relation. Reification is "a general purpose technique for representing N-ary relations using a language such as OWL that permits only binary relations" (Batsakis and Petrakis 2011). Versioning approach is described as "the ability to handle changes in ontologies by creating and managing different variants of it" (Klein and Fensel 2001). When an ontology is modified, a new version is created to represent the temporal evolution of the ontology. 4D-fluents approach represents temporal information and the evolution of the last ones in OWL. Concepts varying in time are represented as 4-dimensional objects with the 4th dimension being the temporal dimension.

Based on the present related work, we choose the 4D-fluents approach. Indeed, compared to related work, it minimizes the problem of data redundancy as the changes occur only on the temporal parts and keeping therefore the static part unchanged. It also maintains full OWL expressiveness and reasoning support (Batsakis and Petrakis

2011). We extend this approach in two ways. (1) It is extended with crisp components to represent imprecise time intervals and crisp interval relations in OWL 2 (Section 3). (2) It is extended with fuzzy components to represent imprecise time intervals and fuzzy interval relations in Fuzzy-OWL 2 (Section 4).

2.2 Allen's Interval Algebra

(Allen 1983) has proposed 13 mutually exclusive primitive relations that may hold between two precise time intervals. Their semantics is illustrated in Table 1. Let $I = [I^-, I^+]$ and $J = [J^-, J^+]$ two time intervals; where I^- (respectively J^-) is the beginning time-step of the event and I^+ (respectively J^-) is the ending.

A number of works fuzzify Allen's temporal interval relations. We classify these works into (1) works focusing on fuzzifying Allen's interval algebra to compare precise time intervals and (2) works focusing on fuzzifying Allen's interval algebra to compare imprecise time intervals.

Three approaches have been proposed to fuzzify Allen's interval algebra in order to compare precise time intervals: (Guesgen et al. 1994), (Dubois and Prade 1989) and (Badaloni and Giacomin 2006). (Guesgen et al. 1994) propose fuzzy Allen relations viewed as fuzzy sets of ordinary Allen relationship taking into account a neighborhood structure, a notion originally introduced in (Freksa 1992). (Dubois and Prade 1989) represent a time interval as a pair of possibility distributions that define the possible values of the endpoints of the crisp interval. Using possibility theory, the possibility and necessity of each of the interval relations can then be calculated. This approach also allows modeling imprecise relations such as "long before". (Badaloni and Giacomin 2006) propose a fuzzy extension of Allen's work, called IAfuz where degrees of preference are associated to each relation between two precise time intervals.

Four approaches have been proposed to fuzzify Allen's interval algebra to compare imprecise time intervals: (Nagypál and Motik 2003), (Ohlbach 2004), Schockaert08 and (Gammoudi et al. 2017). (Nagypál and Motik 2003) propose a temporal model based on fuzzy sets to extend Allen relations with imprecise time intervals. The authors introduce a set of auxiliary operators on intervals and define fuzzy counterparts of these operators. The compositions of these relations are not studied by the authors. (Ohlbach 2004) propose an approach to handle some gradual temporal relations as "more or less finishes". However, this work cannot take into account gradual temporal relations such as "long before". Furthermore, many of the symmetry, reflexivity, and transitivity properties of the original temporal interval relations are lost in this approach; thus it is not suitable for temporal reasoning. (Schockaert and Cock 2008) propose a generalization of Allen's relations with precise and imprecise time intervals. This approach allows handling classical temporal relations, as well as other imprecise relations. Interval relations are defined according to two fuzzy operators comparing two time instants: "long before" and "occurs before or at approximately the same time". (Gammoudi et al. 2017) generalize the definitions of the 13 Allen's classic interval relations to make them applicable to fuzzy intervals in two ways (conjunctive and disjunctive). Gradual temporal interval relations are not taken into account.

3 A Crisp-Based Approach for Representing and Reasoning on Imprecise Time Intervals

In this section, we propose a crisp-based approach to represent and reason on imprecise time intervals. This solution is entirely based on crisp standards and tools. We extend the 4D-fluents model to represent imprecise time intervals and their crisp relationships in OWL 2. To reason on imprecise time intervals, we extend the Allen's interval algebra in a crisp way. In OWL 2, inferences are done via a set of SWRL rules.

3.1 Representing Imprecise Time Intervals and Crisp Qualitative Interval Relations in OWL 2

In the crisp-based solution, we now represent each imprecise interval bound of the time interval as

Table 1: Allen's temporal interval relations (I: , J:).

Relation	Inverse	Relations between interval bounds	Illustration
$Before(I,J)$	$After(I,J)$	$I^+ < J^-$	
$Meets(I,J)$	$MetBy(I,J)$	$I^+ = J^-$	
$Overlaps(I,J)$	$OverlappedBy(I,J)$	$(I^- < J^-) \wedge (I^+ > J^-) \wedge (I^+ < J^+)$	
$Starts(I,J)$	$StartedBy(I,J)$	$(I^- = J^-) \wedge (I^+ < J^+)$	
$During(I,J)$	$Contains(I,J)$	$(I^- > J^-) \wedge (I^+ < J^+)$	
$Ends(I,J)$	$EndedBy(I,J)$	$(I^- > J^-) \wedge (I^+ = J^+)$	
$Equal(I,J)$	$Equal(I,J)$	$(I^- = J^-) \wedge (I^+ = J^+)$	

a disjunctive ascending set. Let $I = [I^-, I^+]$ be an imprecise time interval; where $I^- = I^{-(1)} \ldots I^{-(N)}$ and $I^+ = I^{+(1)} \ldots I^{+(N)}$. For instance, if we have the information "Alexandre was started his PhD study in 1975 and he was graduated around 1980" the imprecise time interval representing this period is [1975, {1978 . . . 1982}]. This means that his PhD studies end in 1978 or 1979 or 1980 or 1981 or 1982. The classic 4D-fluents model introduces two crisp classes "TimeSlice" and "TimeInterval" and four crisp properties "tsTimeSliceOf", "tsTimeInterval", "hasBegining" and "hasEnd". The class "TimeSlice" is the domain class for entities representing temporal parts (i.e., "time slices"). The property "tsTimeSliceOf" connects an instance of class "TimeSlice" with an entity. The property "tsTimeInterval" connects an instance of class "TimeSlice" with an instance of class "TimeInterval". The instance of class "TimeInterval" is related with two temporal instants that specify its starting and ending points using, respectively, the "hasBegining" and "hasEnd" properties. Figure 1 illustrates the use of the 4D-fluents model to represent the following example: "Alexandre was started his PhD study in 1975 and he was graduated in 1978".

We extend the original 4D-fluents model in the following way. We add four crisp datatype properties "HasBeginningFrom", "HasBeginningTo", "HasEndFrom", and "HasEndTo" to the class "TimeInterval". Let $I = [I^-, I^+]$ be an imprecise time interval; where $I^- = I^{-(1)} \ldots I^{-(N)}$ and $I^+ = I^{+(1)} \ldots I^{+(N)}$. "HasBeginningFrom" has the range $I^{-(1)}$. "HasBeginningTo" has the range $I^{-(N)}$. "HasEndFrom" has the range $I^{+(1)}$. "HasEndTo" has the range $I^{+(N)}$. The 4D-fluents model is also enhanced with crisp qualitative temporal interval relations that may hold between imprecise time intervals. This is implemented by introducing temporal relationships, called "RelationIntervals", as a crisp object property between two instances of the class "TimeInterval". Figure 2 represents the extended 4D-fluents model in OWL 2.

We can see in Figure 3 an instantiation of the extended 4D-fluents model in OWL 2. On this example, we consider the following information: "Alexandre was married to Nicole just after he was graduated with a PhD. Alexandre was graduated with a PhD in 1980. Their marriage lasts 15 years. Alexandre was remarried to Béatrice since about 10 years and they were divorced in 2016". Let $I = [I^-, I^+]$ and $J = [J^-, J^+]$ be two imprecise time intervals representing, respectively, the duration of the marriage of Alexandre with Nicole and the one with Béatrice. Assume that $I^- = \{1980 \ldots 1983\}$, $I^+ = \{1995 \ldots 1998\}$, $J^- = \{2006 \ldots 2008\}$ and $J^+ = 2016$.

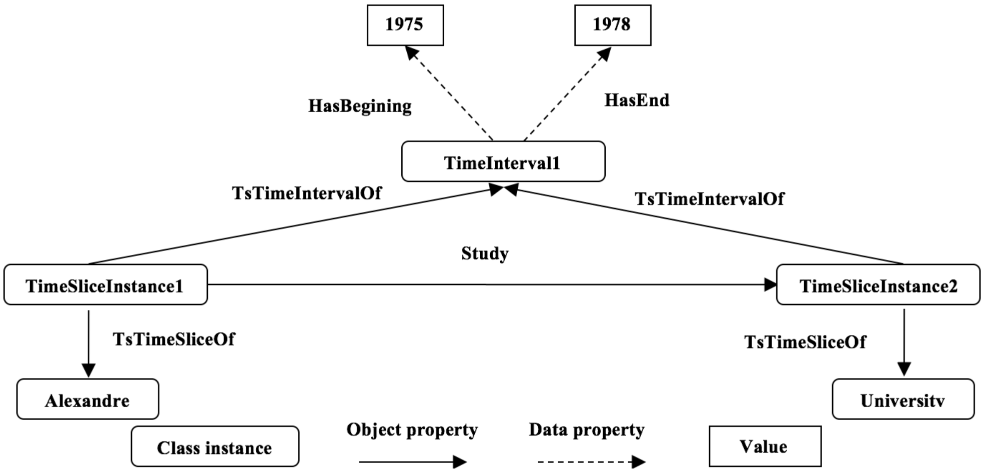

Figure 1: An instantiation of the classic the 4D-fluents model.

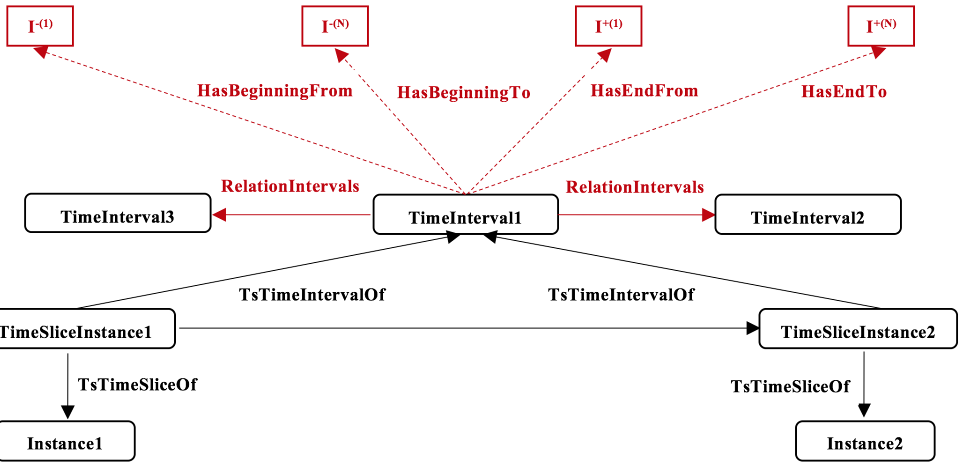

Figure 2: The extended 4D-fluents model in OWL 2.

3.2 A Crisp-Based Reasoning on Imprecise Time Intervals in OWL 2

We have redefined 13 Allen's interval, in a crisp way, to compare imprecise time intervals i.e., the resulting interval relations upon imprecise time intervals are crisp. Let $I = [I^-, I^+]$ and $J = [J^-, J^+]$ two imprecise time intervals; where $I^- = I^{-(1)} \ldots I^{-(N)}$, $I^+ = I^{+(1)} \ldots I^{+(N)}$, $J^- = J^{-(1)} \ldots J^{-(N)}$ and $J^+ = J^{+(1)} \ldots J^{+(N)}$. For instance, the crisp interval relation "before (I, J)" is redefined as: $\forall I^{+(i)} \in I^+, \forall J^{-(j)} \in J^- : I^{+(i)} < J^{-(j)}$ This means that the most recent time instant of $I^+ (I^{+(N)})$ ought to Precede the oldest time instant of $J^- (J^{-(1)})$: $I^{+(N)} < J^{-(1)}$ In the similar way, we define the other temporal interval relations. In Table 2, we define 13 crisp temporal interval relations upon the two imprecise time intervals I and J.

In order to apply our crisp extension of Allen's work in OWL 2, we propose a set of SWRL rules that infer the temporal interval relations from the introduced imprecise time intervals which are represented using the extended 4D-fluents model in OWL2. For each temporal interval relation, we associate a SWRL rule. Reasoners that support DL-safe rules (i.e., rules that apply only on named individuals in the knowledge base) such as Pellet (Sirin et al. 2007) can support our approach. For

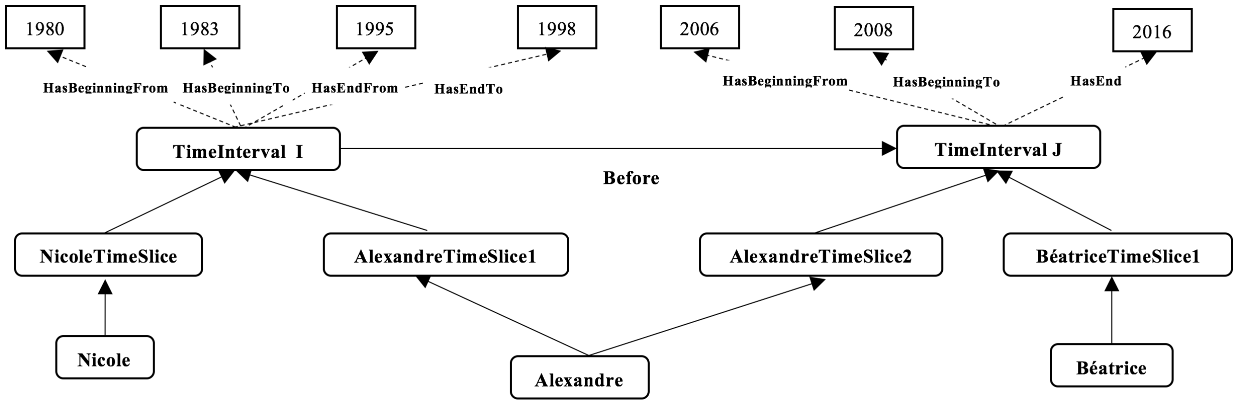

Figure 3: An instantiation of the extended 4D-fluents model in OWL 2.

instance, the SWRL rule to infer the "Meet (I, J)" relation is the following:

$$TimeInterval(I) \wedge TimeInterval(J) \wedge HasEndFrom(I, a) \wedge HasBeginningFrom(J, b) \wedge Equals(a, b) \wedge HasEndTo(I, c) \wedge HasBeginningTo(J, d) \wedge Equals(c, d) \rightarrow Meet(I, J)$$

4 A Fuzzy-Based Approach for Representing and Reasoning on Imprecise Time Intervals

In this section, we propose a fuzzy-based approach to represent and reason on imprecise time intervals. This approach is based on a fuzzy environment. We extend the 4D-fluents model to represent imprecise time intervals and their relationships in Fuzzy-OWL 2. To reason on imprecise time intervals, we extend the Allen's interval algebra in a fuzzy gradual personalized way. We infer the resulting fuzzy interval relations in Fuzzy-OWL 2 using a set of Mamdani IF-THEN rules.

4.1 Representing Imprecise Time Intervals and Fuzzy Qualitative Interval Relations in Fuzzy-OWL 2

In the fuzzy-based solution, we now represent the imprecise beginning interval bound as a fuzzy set which has the L-function MF and the ending interval bound as a fuzzy set which has the R-function membership function (MF). Let $I = [I^-, I^+]$ be an imprecise time interval. We represent the binging bound I^- as a fuzzy set which has the L-function MF ($A = I^{-(1)}$ and $B = I^{-(N)}$). We represent the ending bound I^+ as a fuzzy set which has the R-function MF ($A = I^{+(1)}$ and $B = I^{+(N)}$). For instance, if we have the information "Alexandre was starting his PhD study in 1973 and was graduated in late 80", the beginning bound is crisp. The ending bound is imprecise and it is represented by L-function MF ($A = 1976$ and $B = 1980$). For the rest of the paper, we use the MFs shown in Figure 4 [Zadeh, 1975].

We extend the original 4D-fluents model to represent imprecise time intervals in the following way. We add two fuzzy datatype properties "FuzzyHasBegining" and "FuzzyHasEnd" to the class "TimeInterval". "FuzzyHasBegining" has the L-function MF ($A = I^{-(1)}$ and $B = I^{-(N)}$). "FuzzyHasEnd" has the R-function MF ($A = I^{+(1)}$ and $B = I^{+(N)}$). The 4D-fluents approach is also enhanced with qualitative temporal relations that may hold between imprecise time intervals. We introduce the "FuzzyRelationIntervals", as a fuzzy object property between two instances of the class "TimeInterval". "FuzzyRelationIntervals" represent fuzzy qualitative temporal relations. "FuzzyRelationIntervals" has the L-function MF ($A = 0$ and $B = 1$). Figure 5 represents our extended 4D-fluents model in Fuzzy-OWL 2.

We can see in Figure 6 an instantiation of the extended 4D-fluents model in Fuzzy-OWL 2. On this

Table 2: Crisp temporal interval relations upon imprecise time intervals.

Relation	Inverse	Interpretation	Relations between interval bounds
$Before(I,J)$	$After(I,J)$	$\forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-} : (I^{+(i)} < J^{-(j)})$	$I^{+(N)} < J^{-(1)}$
$Meets(I,J)$	$MetBy(I,J)$	$\forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-} : (I^{+(i)} = J^{-(j)})$	$(I^{+(1)} = J^{-(1)}) \wedge (I^{+(N)} = J^{-(N)})$
$Overlaps(I,J)$	$OverlappedBy(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(j)} \in J^{+} : (I^{-(i)} < J^{-(j)}) \wedge (J^{-(j)} < I^{+(i)}) \wedge (I^{+(i)} < J^{+(j)})$	$(I^{-(N)} < J^{-(1)}) \wedge (J^{-(N)} < I^{+(1)}) \wedge (I^{+(N)} < J^{+(1)})$
$Starts(I,J)$	$StartedBy(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(j)} \in J^{+} : (I^{-(i)} = J^{-(j)}) \wedge (I^{+(i)} < J^{+(j)})$	$(I^{-(1)} = J^{-(1)}) \wedge (I^{-(N)} = J^{-(N)}) \wedge (I^{+(N)} < J^{+(1)})$
$During(I,J)$	$Contains(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(i)} \in J^{+} : (J^{-(j)} < I^{-(i)}) \wedge (I^{+(i)} < J^{+(i)})$	$(J^{-(N)} < I^{-(1)}) \wedge (I^{+(N)} < J^{+(1)})$
$Ends(I,J)$	$EndedBy(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(j)} \in J^{+} : (I^{-(i)} < J^{-(j)}) \wedge (I^{+(i)} = J^{+(j)})$	$(J^{-(N)} < I^{-(1)}) \wedge (I^{+(1)} = J^{+(1)}) \wedge (I^{+(N)} = J^{+(N)})$
$Equal(I,J)$	$Equal(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(j)} \in J^{+} : (I^{-(i)} = J^{-(j)}) \wedge (I^{+(i)} = J^{+(j)})$	$(I^{-(1)} = J^{-(1)}) \wedge (I^{-(N)} = J^{-(N)}) \wedge (I^{+(1)} = J^{+(1)}) \wedge (I^{+(N)} = J^{+(N)})$

example, we consider the following information: "Alexandre was married to Nicole just after he was graduated with a PhD. Alexandre was graduated with a PhD in 1980. Their marriage lasts 15 years. Alexandre was remarried to Béatrice since about 10 years and they were divorced in 2016". Let $I = [I^{-}, I^{+}]$ and $J = [J^{-}, J^{+}]$ be two imprecise time intervals representing, respectively, the duration of the marriage of Alexandre with Nicole and the one with Béatrice. I^{-} is represented with the fuzzy datatype property "FuzzyHasBegining" which has the L-function MF ($A = 1980$ and $B = 1983$). I^{+} is represented with the fuzzy datatype property "FuzzyHasEnd" which has the R-function MF ($A = 1995$ and $B = 1998$). J^{-} is represented with the fuzzy datatype property "FuzzyHasBegining" which has the L-function MF ($A = 2005$ and $B = 2007$). J^{+} is represented with the crisp datatype property "HasEnd" which has the value "2016".

4.2 A Fuzzy-Based Reasoning on Imprecise Time Intervals in Fuzzy OWL 2

We propose a set of fuzzy gradual personalized comparators that may hold between two time instants. Based on these operators, we present our fuzzy gradual personalized extension of Allen's work. Then, we infer, in Fuzzy OWL 2, the resulting temporal interval relations via a set of Mamdani IF-THEN rules using the fuzzy reasoner FuzzyDL. We generalize the crisp time instants comparators "Follow", "Precede" and "Same", introduced in (Vilain and Kautz 1986). Let α and β two parameters allowing the definition of the membership function of the following comparators ($\in]0, +\infty[$); N is the number of slices; T_1 and T_2 are two time instants; we define the following comparators (illustrated in Figure 7):

- $\{Follow_{(1)}^{(\alpha,\beta)}(T_1, T_2) \ldots Follow_{(N)}^{(\alpha,\beta)}(T_1, T_2)\}$ are a generalization of the crisp time instants relation "Follows". $Follow_{(1)}^{(\alpha,\beta)}(T_1, T_2)$ means that T_1 is "just after or approximately at the same time" T_2 w.r.t. (α, β) and gradually the time gap between T_1

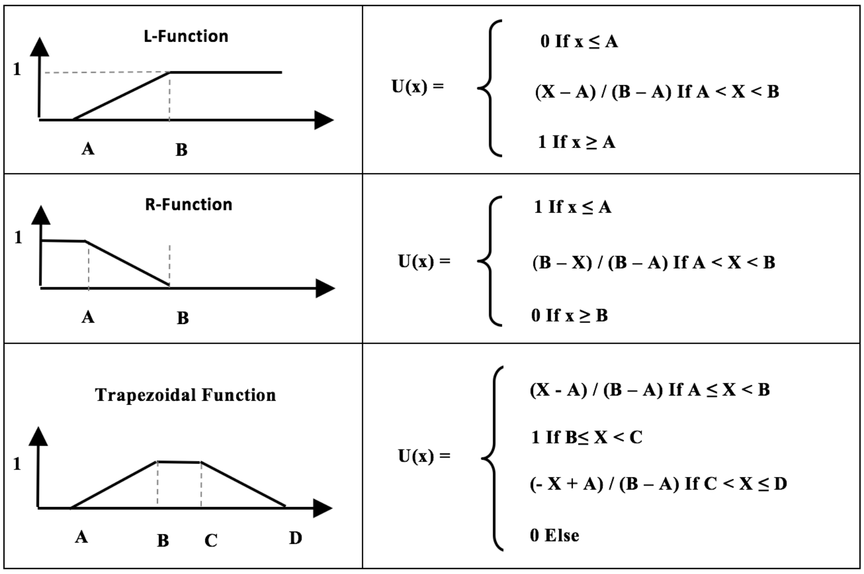

Figure 4: R-Function, L-Function and Trapezoidal MFs (Zadeh 1975).

and T_2 increases until $Follow_{(N)}^{(\alpha,\beta)}(T_1,T_2)$ which means that T_1 is "long after" T_2 w.r.t. (α,β). N is set by the expert domain. $\{Follow_{(1)}^{(\alpha,\beta)}(T_1,T_2)\ldots Follow_{(N)}^{(\alpha,\beta)}(T_1,T_2)\}$ are defined as fuzzy sets. $Follow_{(1)}^{(\alpha,\beta)}(T_1,T_2)$ has R-Function MF which has as parameters $A=\alpha$ and $B=(\alpha+\beta)$. All comparators $\{Follow_{(2)}^{(\alpha,\beta)}(T_1,T_2)\ldots Follow_{(N-1)}^{(\alpha,\beta)}(T_1,T_2)\}$ have trapezoidal MF which has as parameters $A=((K-1)\alpha)$ and $B=((K-1)\alpha+(K-1)\beta)$, $C=(K\alpha+(K-1)\beta)$ and $D=(K\alpha+K\beta)$; where $2\leq K\leq N-1$. $Follow_{(N)}^{(\alpha,\beta)}(T_1,T_2)$ has L-Function MF which has as parameters $A=((N-1)\alpha+(N-1)\beta)$ and $B=((N-1)\alpha+(N-1)\beta)$;

- $\{Precede_{(1)}^{(\alpha,\beta)}(T_1,T_2)\ldots Precede_{(N)}^{(\alpha,\beta)}(T_1,T_2)\}$ are a generalization of the crisp time instants relation "Precede". $Precede_{(1)}^{(\alpha,\beta)}(T_1,T_2)$ means that T_1 is "just before or approximately at the same time" T_2 w.r.t. (α,β) and gradually the time gap between T_1 and T_2 increases until $Precede_{(N)}^{(\alpha,\beta)}(T_1,T_2)$ which means that T_1 is "long before" T_2 w.r.t. (α,β). N is set by the expert domain. $\{Precede_{(1)}^{(\alpha,\beta)}(T_1,T_2)\ldots Precede_{(N)}^{(\alpha,\beta)}(T_1,T_2)\}$ are defined as fuzzy sets. $Precede_{(i)}^{(\alpha,\beta)}(T_1,T_2)$ is defined as:

$$Precede_{(i)}^{(\alpha,\beta)}(T_1,T_2)=1-Follow_{(i)}^{(\alpha,\beta)}(T_1,T_2)$$

- We define the comparator $Same^{(\alpha,\beta)}$ which is a generalization of the crisp time instants relation "Same". $Same^{(\alpha,\beta)}(T_1,T_2)$ means that T_1 is "approximately at the same time" T_2 w.r.t. (α,β). It is defined as:

$$Same^{(\alpha,\beta)}(T_1,T_2)=Min(Follow_{(1)}^{(\alpha,\beta)}(T_1,T_2),\ Precede_{(1)}^{(\alpha,\beta)}(T_1,T_2))$$

Then, we extend Allen's work to compare imprecise time intervals with a fuzzy gradual personalized view. We provide a way to model gradual, linguistic-like description of temporal interval relations. Compared to related work, our work is not limited to a given number of imprecise relations. It is possible to determinate the level of precision that should be in a given context. For instance, the classic Allen relation "before" may be generalized in N imprecise relations, where "$Before_{(1)}^{(\alpha,\beta)}(I,J)$" means that I is "just before" J w.r.t. (α,β) and gradually the time gap between I and J increases until "$Before_{(N)}^{(\alpha,\beta)}(I,J)$" which

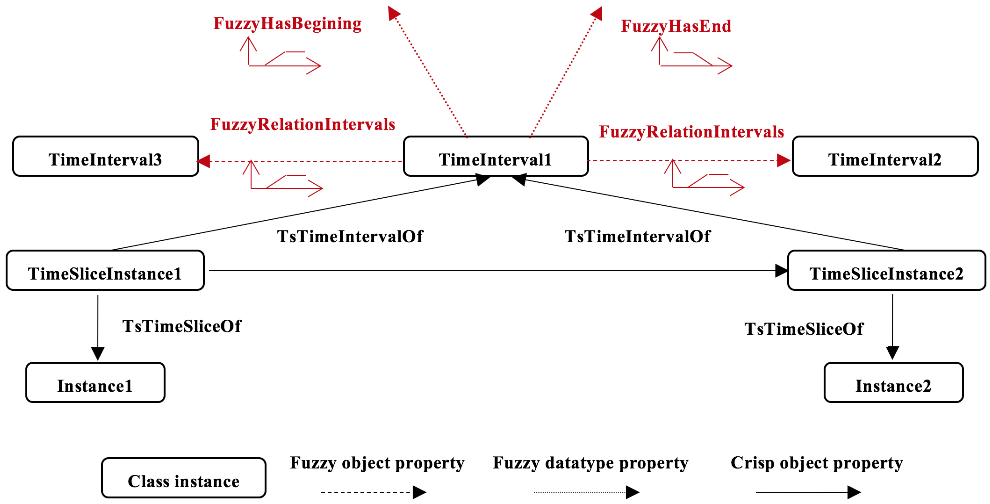

Figure 5: The extended 4D-fluents model in Fuzzy-OWL 2.

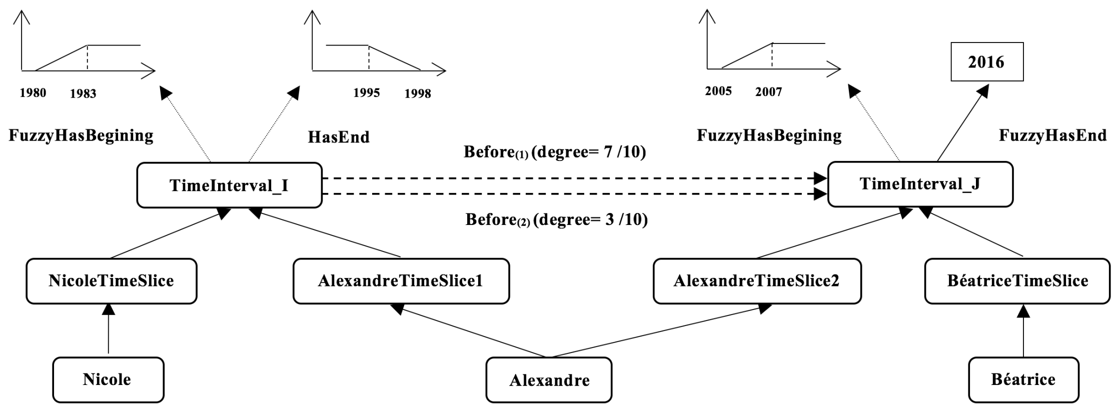

Figure 6: An instantiation of the extended 4D-fluents model in Fuzzy-OWL 2.

means that I is long before J w.r.t. (α, β). The definition of our fuzzy interval relations is based on the fuzzy gradual personalized time instants compactors. Let $I = [I^-, I^+]$ and $J = [J^-, J^+]$ two imprecise time intervals; where I^- has the L-function MF ($A = I^{-(1)}$ and $B = I^{-(N)}$); I^+ is a fuzzy set which has the R-function MF ($A = I^{+(1)}$ and $B = I^{+(N)}$); J^- is a fuzzy set which has the L-function MF ($A = J^{-(1)}$ and $B = J^{-(N)}$); J^+ is a fuzzy set which has the R-function MF ($A = J^{+(1)}$ and $B = J^{+(N)}$). For instance, the fuzzy interval relation "$Before_{(1)}^{(\alpha,\beta)}(I, J)$" is defined as:

$$\forall I^{+(i)} \in I^+, \forall J^{-(j)} \in J^- : \quad Precede_{(1)}^{(\alpha,\beta)}(I^{+(i)}, J^{-(j)})$$

This means that the most recent time instant of $I^+(I^{+(N)})$ ought to proceed the oldest time instant of $J^-(J^{-(1)})$:

$$Precede_{(1)}^{(\alpha,\beta)}(I{+}(N), J{-}(1))$$

In the similar way, we define the others temporal interval relations, as shown in Table 3.

Finally, we have implemented our fuzzy gradual personalized extension of Allen's work in Fuzzy-OWL 2. We use the ontology editor PROTEGE version 4.3 and the fuzzy reasoner FuzzyDL. We propose a set of Mamdani IF-THEN rules to infer the temporal interval relations from the introduced imprecise time intervals which are represented using the extended 4D-fluents model in

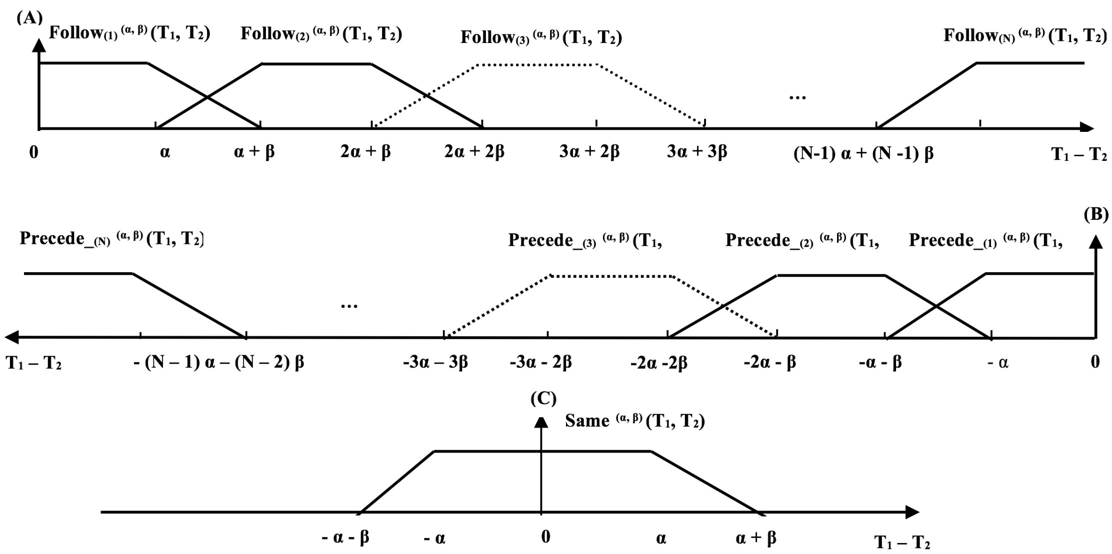

Figure 7: Fuzzy gradual personalized time instants comparators. (A) Fuzzy sets of $\{Follow^{(\alpha,\beta)}_{(1)}(T_1,T_2)\ldots Follow^{(\alpha,\beta)}_{(N)}(T_1,T_2)\}$. (B) Fuzzy sets of $\{Precede^{(\alpha,\beta)}_{(1)}(T_1,T_2)\ldots Precede^{(\alpha,\beta)}_{(N)}(T_1,T_2)\}$. (C) Fuzzy set of $Same^{(\alpha,\beta)}(T_1,T_2)$.

Table 3: Fuzzy gradual personalized temporal interval relations upon imprecise time intervals.

Relation	Inverse	Relations between bounds	Definition
$Before^{(\alpha,\beta)}_{(K)}(I,J)$	$After^{(\alpha,\beta)}_{(K)}(I,J)$	$\forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}: (I^{+(i)} < J^{-(j)})$	$Precede^{(\alpha,\beta)}_{(K)}(I^{+(N)}, J^{-(1)})$
$Meets^{(\alpha,\beta)}(I,J)$	$MetBy^{(\alpha,\beta)}(I,J)$	$\forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}: (I^{+(i)} = J^{-(j)})$	$Min(Same^{(\alpha,\beta)}(I^{+(1)}, J^{-(1)}) \wedge Same^{(\alpha,\beta)}(I^{+(N)}, J^{-(N)}))$
$Overlaps^{(\alpha,\beta)}_{(K)}(I,J)$	$OverlappedBy^{(\alpha,\beta)}_{(K)}(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(j)} \in J^{+}: (I^{-(i)} < J^{-(j)}) \wedge (J^{-(j)} < I^{+(i)}) \wedge (I^{+(i)} < J^{+(j)})$	$Min(Precede^{(\alpha,\beta)}_{(K)}(I^{-(N)}, J^{-(1)}) \wedge Precede(K)^{(\alpha,\beta)}(J^{-(N)}, I^{+(1)}) \wedge Precede^{(\alpha,\beta)}_{(K)}(I^{+(N)}, J^{+(1)}))$
$Starts^{(\alpha,\beta)}_{(K)}(I,J)$	$StartedBy^{(\alpha,\beta)}_{(K)}(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(j)} \in J^{+}: (I^{-(i)} = J^{-(j)}) \wedge (I^{+(i)} < J^{+(j)})$	$Min(Same^{(\alpha,\beta)}(I^{-(1)}, J^{-(1)}) \wedge Same^{(\alpha,\beta)}(I^{-(N)}, J^{-(N)}) \wedge Precede^{(\alpha,\beta)}_{(K)}(I^{+(N)}, J^{+(1)}))$
$During^{(\alpha,\beta)}_{(K)}(I,J)$	$Contains^{(\alpha,\beta)}_{(K)}(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(i)} \in J^{+}: (J^{-(j)} < I^{-(i)}) \wedge (I^{+(i)} < J^{+(i)})$	$Min(Precede^{(\alpha,\beta)}_{(K)}(J^{-(N)}, I^{-(1)}) \wedge Precede^{(\alpha,\beta)}_{(K)}(I^{+(N)}, J^{+(1)}))$
$Ends^{(\alpha,\beta)}_{(K)}(I,J)$	$EndedBy^{(\alpha,\beta)}_{(K)}(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(j)} \in J^{+}: (I^{-(i)} < J^{-(j)}) \wedge (I^{+(i)} = J^{+(j)})$	$Min(Precede^{(\alpha,\beta)}_{(K)}(J^{-(N)}, I^{-(1)}) \wedge Same^{(\alpha,\beta)}(I^{+(1)}, J^{+(1)}) \wedge Same^{(\alpha,\beta)}(I^{+(N)}, J^{+(N)}))$
$Equal^{(\alpha,\beta)}(I,J)$	$Equal^{(\alpha,\beta)}(I,J)$	$\forall I^{-(i)} \in I^{-}, \forall I^{+(i)} \in I^{+}, \forall J^{-(j)} \in J^{-}, \forall J^{+(j)} \in J^{+}: (I^{-(i)} = J^{-(j)}) \wedge (I^{+(i)} = J^{+(j)})$	$Min(Same^{(\alpha,\beta)}(I^{-(1)}, J^{-(1)}) \wedge Same^{(\alpha,\beta)}(I^{-(N)}, J^{-(N)}) \wedge Same^{(\alpha,\beta)}(I^{+(1)}, J^{+(1)}) \wedge Same^{(\alpha,\beta)}(I^{+(N)}, J^{+(N)}))$

Fuzzy-OWL2. For each temporal interval relation, we associate a Mamdani IF-THEN rule. For instance, the Mamdani IF-THEN rule to infer the "$Overlaps_{(1)}^{(\alpha,\beta)}(I, J)$" relation is the following:

(define-concept Rule0 (g-and (some $Precede_{(1/1)}$ Fulfilled) (some $Precede_{(1/2)}$ Fulfilled) Fulfilled) (some $Precede_{(1/3)}$ Fulfilled) (some $Overlaps_{(1)}$ True))) // Fuzzy rule

We define three input fuzzy variables, named "Precede(1/1)", "Precede(1/2)" and "Precede(1/3)", which have the same MF than that of "$Precede_{(1)}^{(\alpha,\beta)}$". We define one output variable "Overlaps(1)" which has the same membership than that of the fuzzy object property "FuzzyRelationIntervals". "Precede(1/1)", "Precede(1/2)" and "Precede(1/3)" are instantiated with, respectively, $(I^{-(N)} - J^{-(1)})$, $(J^{-(N)} - I^{+(1)})$ and $(I^{+(N)} - J^{+(1)})$.

5 Conclusion

In this paper, we proposed two approaches to represent and reason on imprecise time intervals in OWL: a crisp-based approach and a fuzzy-based approach. (1) The crisp-based approach is based only on crisp environment. We extended the 4D-fluents model to represent imprecise time intervals and crisp interval relations in OWL 2. To reason on imprecise time intervals, we extended the Allen's interval algebra in a crisp way. Inferences are done via a set of SWRL rules. (2) The fuzzy-based approach is entirely based only on fuzzy environment. We extended the 4D-fluents model to represent imprecise time intervals and fuzzy interval relations in Fuzzy-OWL 2. To reason on imprecise time intervals, we extend the Allen's interval algebra in a fuzzy gradual personalized way. We infer the resulting fuzzy interval relations in Fuzzy-OWL 2 using a set of Mamdani IF-THEN rules.

Concerning the choice between these two approaches (the crisp-based one or the fuzzy-based one), as a fuzzy ontology is an extension of crisp ontology, researchers may choose any of our two approaches for introducing imprecise interval management in their knowledge bases whatever these latter are crisp or fuzzy. However our fuzzy-based approach allows more functionalities, in particular it is suitable to represent and reason on gradual interval relations such as "middle before" or "approximately at the same time". Hence, in the case of manipulating a fuzzy knowledge base, we encourage researchers to choose the fuzzy-based approach to model and reason on imprecise time intervals. The main interest of the crisp-based approach is that this solution can be implemented with classical crisp tools and that the programmers are not obliged to learn technologies related to fuzzy ontology. Considering that crisp tools and models are more mature and better support scaling, the crisp-based approach is more suitable for marketed products.

The works presented in this paper have been tested in two projects, having in common to manage life logging data: (1) In the VIVA project, we aim to design the Captain Memo memory prosthesis (Herradi et al. 2015; Métais et al. 2012) for Alzheimer Disease patients. Among other functionalities, this prosthesis manages a knowledge base on the patient's family tree, using an OWL ontology. Imprecise inputs are especially numerous when given by an Alzheimer Disease patient. Furthermore, dates are often given in reference to other dates or events. Thus we have been using the "fuzzy" solution reported in this paper. One interesting point in this solution is to deal with a personalized slicing of the person's life in order to sort the different events. (2) The QUALHIS project aims to allow Middle Ages specialized historians to deal with prosopographical data bases storing Middle age academic's career histories. Data come from various archives among Europe and data about a same person are very difficult to align. Representing and ordering imprecise time interval is required to redraw the careers across the different European universities who hosted the person. We preferred the "crisp" solution in order to favour the integration within the existing crisp ontologies and tools, and to ease the management by Historians.

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