Asynchronous OFDM/FBMC Interference Analysis in Selective Channels

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***Abstract*—This paper addresses the effect of timing synchronization errors for two different multicarrier techniques CP-OFDM and FBMC. Such errors degrade the performance of the reference receiver by causing multi user interference. This interference occurs when the signals from different transmitters arrive at a given receiver with arbitrary timing misalignments, leading to the destruction of orthogonality between subcarriers. In this paper, we propose a model of interference that provides a useful computational tool in order to analyze the performance of an OFDM/FBMC receiver in frequency selective fading environment. Finally, simulation results confirmed the accuracy of the proposed model.**

***Index Terms*—Interference, OFDM, FBMC, timing non-synchronization, selective frequency channel.** [1]

I. Introduction

Interference at the radio receiver is a key source of degradation in quality of service of wireless communication systems. Multicarrier based systems do not suffer from interference among multiplexed users within a cell given perfect synchronization. However, in some cases such as cognitive radio, non cooperative base stations or Ad-hoc networks, it is very difficult to maintain the synchronization. Consequently, it is relevant to evaluate the impact of the asynchronous interference on the system's performance.

Several models have been proposed to investigate this problem such as the classical Gaussian approximation (GA) techniques [1],[2]; and the power spectral density (PSD) modeling [3],[4],[5]. In some cases, the interference is correlated because the accumulated interference is no longer a sum of independent RVs. In these cases, the GA model is inaccurate [6]. Furthermore, the asynchronous interference depends mainly on the time synchronization errors while the PSD model gives the same result for all possible timing errors.

In [7], a new model of inter-cell interference has been proposed for two multicarrier modulation techniques : Orthogonal Frequency Division Multiplexing using the Cyclic prefix (CP-OFDM) and the Filter Bank based Multi-Carrier (FBMC) waveform. It is based on the computation of the interference power at the output of the receiver filter corresponding to a given timing offset. Two tables, modeling the mean interference power, are given for timing offset uniformly distributed on the OFDM block duration. It is worth noting that these tables have been evaluated for a constant unitary channel.

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The purpose of this paper is to analyze the asynchronous interference in frequency selective environment. CP-OFDM with a rectangular waveform and FBMC with a time-frequency localized shaping pulse [8] are considered. We propose to model the interference by an instantaneous interference table for each timing offset instead of the mean interference table calculated in [7]. The exactness of this model will be discussed considering pathloss and selective Rayleigh fading effects with respect to two criteria: the signal to interference plus noise ratio (SINR) and the user capacity. A comparison with similar results, obtained through intensive Monte Carlo simulations, is presented.

The outline of this paper is organized as follows. In Section II, we briefly illustrate the theoretical derivation of the instantaneous table and we analyze also theoretically the asynchronous interference in the presence of selective rayleigh fading. In Section III, we propose an interference estimation method using the instantaneous tables. Simulation results are reported in Section IV, and Section V concludes this paper.

II. Asynchronous OFDM/FBMC transmission over a frequency selective channel

First of all, we illustrate the theoretical derivation of the instantaneous interference tables for CP-OFDM and FBMC systems. These tables are computed considering an asynchronous transmission over a constant unitary channel.

A. Instantaneous Interference Tables

We refer to a reference receiver which suffers from interference coming from an asynchronous source with a given timing offset τ.

Following [7], the asynchronous OFDM interference signal received on the k'^{th} subcarrier, considering the transmission of a single symbol $x_{k,0}$ on the k^{th} subcarrier

$$y_{k'}(\tau) = x_{k,0} e^{-j\frac{2\pi}{T}k\tau} \times \begin{cases} \delta(l) & \tau \in [0, \Delta] \\ e^{j\frac{\pi l}{T}(T+\tau+\Delta)} \frac{sin(\pi l(T+\Delta-\tau)/T)}{\pi l} & \tau \in [\Delta, T+\Delta] \end{cases} \quad (1)$$

where

- $l = k - k'$ and $\delta(l)$ is the Kronecker delta function
- T is the OFDM symbol duration
- Δ is the cyclic prefix duration

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In general case, the resulting interference power is the sum of interference powers coming resp. from two successive data symbols $(x_{k,n-1}, x_{k,n})$, we get then

$$I(\tau, l) = \begin{cases} \delta(l) & \tau \in [0, \Delta] \\ \left|\frac{\sin(\pi l(T+\Delta-\tau)/T)}{\pi l}\right|^2 + \left|\frac{\sin(\pi l(\tau-\Delta)/T)}{\pi l}\right|^2 & \tau \in [\Delta, T+\Delta] \end{cases} \quad (2)$$

It should be noted that the data communication symbols $x_{k,n}$ are zero mean uncorrelated variables with a power of 1.

On the other hand, transmit pulses, that are more localized in time-frequency domain, are used in the FBMC system [8],[9]. The orthogonality between subcarriers is maintained by introducing half a symbol period between the in-phase and the quadrature components of each complex symbol [9],[10]. This technique is called offset QAM (OQAM) technique.

Now, let us consider the asynchronous transmission of a single symbol $a_{k,n}$ from the interferer to the reference user on the k^{th} subcarrier

$$s(t-\tau) = a_{k,n} g(t - nT/2 - \tau) e^{j\frac{2\pi}{T}k(t-\tau)} e^{j\varphi_{k,n}} \quad (3)$$

where $g(t)$ is the real-valued symmetric pulse response of the prototype filter and $\varphi_{k,n} = \frac{\pi}{2}(n+k) - \pi nk$. T is the signaling interval and $1/T$ is the subcarrier spacing.
The interference signal on the k'^{th} subcarrier of the reference receiver output can be written

$$y_{n',k'}(\tau) = a_{k,n} e^{j(\varphi_{k,n}-\varphi_{k',n'})} e^{-j\frac{2\pi}{T}k\tau} \times \underbrace{\int_{t_1}^{t_2} g(t - nT/2 - \tau) g(t - n'T/2) e^{j\frac{2\pi}{T}lt} dt}_{\Psi(t,\tau,l)\big|_{t=t_1}^{t_2}} \quad (4)$$

In our analysis, we refer to the PHYDYAS NPR (nearly perfect reconstruction) prototype filter using the frequency sampling technique [8]. The overlapping factor K is equal to 4 which means that the impulse response $g(t)$ is non zero when $t \in [0, 4T]$. Therefore, we rewrite (4) as follows

case 1: $(-4T < (n'-n)\frac{T}{2} < \tau)$

$$y_{n',k'}(\tau) = a_{k,n} e^{j(\varphi_{k,n}-\varphi_{k',n'})} e^{-j\frac{2\pi}{T}k\tau} \times \Psi(t,\tau,l)\Big|_{t=\tau}^{4T+(n'-n)} \quad (5)$$

case 2: $(\tau < (n'-n)\frac{T}{2} < 4T)$

$$y_{n',k'}(\tau) = a_{k,n} e^{j(\varphi_{k,n}-\varphi_{k',n'})} e^{-j\frac{2\pi}{T}k\tau} \times \Psi(t,\tau,l)\Big|_{t=(n'-n)\frac{T}{2}}^{4T+\tau} \quad (6)$$

Fig. 1 and 2 show the interference power caused by different timing offsets $\tau = \{\frac{T}{4}, \frac{T}{3}, \frac{T}{2}\}$. In CP-OFDM, we have considered a CP duration of $T/8$. It is worth noting that we can model this interference power as a set of instantaneous interference tables for each timing offset τ.

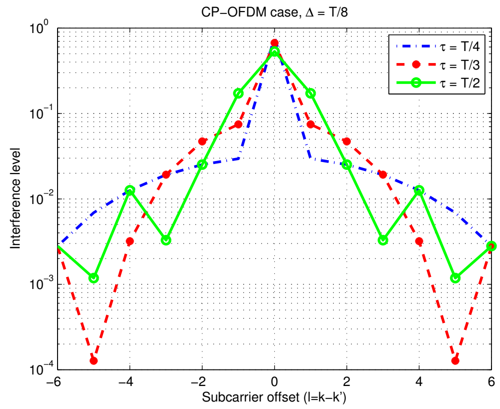

Fig. 1. The interference level in CP-OFDM for $\tau = \{\frac{T}{4}, \frac{T}{3}, \frac{T}{2}\}$

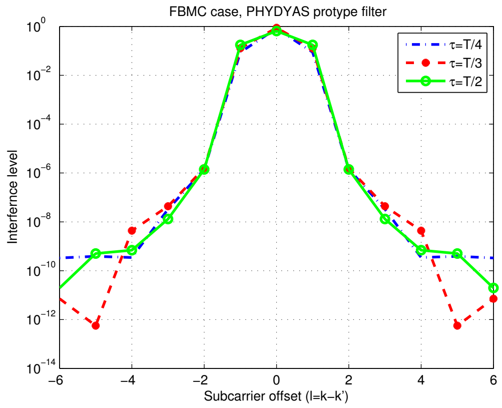

Fig. 2. The interference level in FBMC for $\tau = \{\frac{T}{4}, \frac{T}{3}, \frac{T}{2}\}$

B. Interference with selective channels

1) CP-OFDM case: In a classical OFDM transmission, we choose the cyclic prefix duration Δ greater than the maximum delay spread of the propagation channel. In such case, the CP transforms the linear convolution channel to a cyclic convolution channel. After the FFT operation at the receiver, we obtain

$$P_{rec}(k) = |H_k|^2 P_{trans}(k) \quad (7)$$

where $P_{trans}(k)$ and $P_{rec}(k)$ denote resp. the transmitted and the received power on the k^{th} subcarrier. H_k is the frequency-domain channel gain at the k^{th} subcarrier.
However, the interferer is not necessarily synchronized with the victim user. Therefore, we cannot use (7) in order to compute the interference power.

To analyze this problem, let us consider the unsynchronized transmission depicted in Fig. 3. The impulse response of the multi-path channel between the interferer and the victim user is defined by

$$h(t) = \sum_{i=0}^{L-1} h_i \delta(t - \frac{n_i}{N}T) \quad (8)$$

where N is the number of subcarriers and $n_0 < n_1 < ... <$

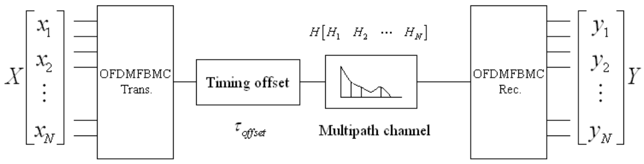

Fig. 3. Unsynchronized OFDM/FBMC transmission

$n_{(L-1)} < C$. C is the maximum delay spread of the channel normalized by the sampling period, and h_i are the complex path gains, which are assumed mutually independent, where $\mathbb{E}[h_i h_i^*] = \gamma_i$, and $\mathbb{E}[h_i h_j^*] = 0$ when $i \neq j$. We further assume that the power is normalized such that $\sum_{i=0}^{L-1} \gamma_i = 1$.

In the following analysis, the channel is assumed stationary over one OFDM symbol. This is the case for time-invariant or slowly varying channels.

Combining the expressions (1) and (8), the interference signal when $\Delta < \tau < T + \Delta$ can be written

$$y_{k'}(\tau) = x_{k,0} e^{-j\frac{2\pi}{T}k\tau} \times \begin{cases} \frac{T+\Delta-\tau}{T} H_k - \frac{1}{N} H'_k & k' = k \\ \frac{1}{j2\pi l}\left(e^{j\frac{2\pi}{T}l\Delta} H_k - e^{j\frac{2\pi}{T}l\tau} H_{k'}\right) & otherwise \end{cases} \quad (9)$$

where H' is the FFT of the following modified channel impulse response

$$h'(t) = \sum_{i=0}^{L-1} n_i h_i \delta(t - \frac{n_i}{N} T) \quad (10)$$

2) FBMC case: For the FBMC case, we refer to the same unsynchronized transmission shown in Fig. 3. Our aim here is to determine the interference caused by an asynchronous FBMC signal in a frequency selective environment.

case 1: $(-4T < (n'-n)\frac{T}{2} < \tau)$

According to the expressions (5) and (8), we obtain

$$y_{n',k'}(\tau) = a_{k,n} e^{j(\varphi_{k,n} - \varphi_{k',n'})} e^{-j\frac{2\pi}{T}k\tau} \times \sum_{i=0}^{L-1} h_i e^{-j\frac{2\pi}{N}kn_i} \, \Psi(t, \tau + \frac{n_i}{N}T, l)\Big|_{t=\tau+\frac{n_i}{N}T}^{4T+(n'-n)\frac{T}{2}} \quad (11)$$

case 2: $(\tau < (n'-n)\frac{T}{2} < 4T)$

In the same way, the expressions (6) and (8) yield

$$y_{n',k'}(\tau) = a_{k,n} e^{j(\varphi_{k,n} - \varphi_{k',n'})} e^{-j\frac{2\pi}{T}k\tau} \times \sum_{i=0}^{L-1} h_i e^{-j\frac{2\pi}{N}mn_i} \, \Psi(t, \tau + \frac{n_i}{N}T, l)\Big|_{t=(n'-n)\frac{T}{2}}^{4T+\tau+\frac{n_i}{N}T} \quad (12)$$

III. Interference estimation using the tables

In the previous section, we have shown that the asynchronous interference in a unitary constant channel depends only on the timing offset τ and the carrier difference between the victim user and the interferer $k - k'$. We have also shown that in frequency selective environments, the interference depends on the two parameters aforementioned and also on the impulse response of the considered propagation channel. Therefore, it is no more possible to generate interference tables because it should be necessary to compute one for each different selective channel. Nevertheless, it is possible to have an estimation of the interference power by using the following expression

$$P_{rec-est}(k', \tau) = P_{trans}(k) I(\tau, |k - k'|) \, |H_k|^2 \quad (13)$$

where

- $P_{trans}(k)$ is the transmitted power on the k^{th} subchannel
- $I(\tau, |k - k'|)$ is the interference table coefficient for the timing offset τ and k' denotes the index of the victim subchannel
- $|H_k|^2$ is the power channel gain on the k^{th} subchannel

Simulation results in Section IV show that the estimated interference using (13) is very closed to the theoretical interference given by (9) in the OFDM case and (11),(12) in the FBMC case.

It is worth noting that we use local interference tables with a length D because the interference is negligible when the interfering subchannel and the victim one are well separated ($|k - k'| > D$). It is clear that the table length D depends on the considered waveform (see Fig. 1 and 2)

IV. Simulation results

In order to assess the accuracy of expression (13), we compare the estimated power using the interference table to the power calculated via Monte-Carlo simulations. We have considered the TU50 model-A as rayleigh fading channel. Channel parameters are given in Table I.

Other simulation parameters involved within this study are summarized in Table II.

Fig. 4 shows the interference power coming from an asynchronous FBMC interferer with a given timing offset τ calculated by the Table estimation (13) and using Monte-Carlo method (over 1000 data frames) for a given channel realization. The channel frequency response is also depicted for this situation. It should be noticed that we assume that the propagation channel is stationary over the whole data frame.

Comparing the Monte-Carlo method (dashed curve) to the Table estimation method (solid curve), we see clearly, that

TABLE I
Channel parameters used in simulations

Parameter	value
TU 50 delays	[0 0.217 0.512 0.514 0.517 0.674 0.882 1.230 1.287 1.311 1.349 1.533 1.535 1.622 1.818 1.836 1.884 1.943 2.048 2.140] ms
TU 50 fading powers	[-5.7 -7.6 -10.1 -10.2 -10.2 -11.5 -13.4 -16.3 -16.9 -17.1 -17.4 -19.0 -19.0 -19.8 -21.5 -21.6 -22.1 -22.6 -23.5 -24.3] dB

TABLE II
STATISTICAL SIMULATION PARAMETERS

Parameter	value	Unit
Total bandwidth B	10	MHz
Bandwidth per subcarrier	9.5	kHz
Center frequency	2.5	GHz
Number of subcarriers	1024	-
Number of subcarriers per cluster	18	-
CP duration Δ	12.8 $(T/8)$	μs
Channel model	TU50 model-A	
Path Loss	128.1+37.6 $Log_{10}(d)$	dB
Thermal noise density	-174 dBm/Hz	
SNR	20	dB

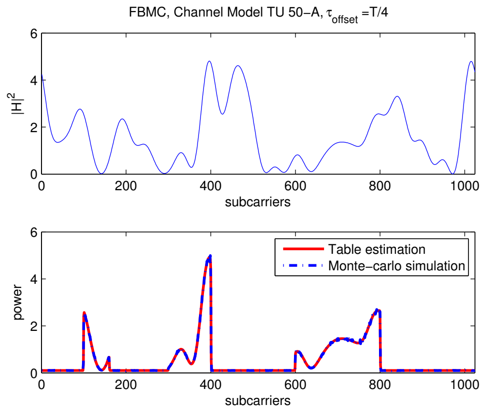

Fig. 4. Table estimation and the Monte-Carlo comparison, FBMC case, Channel Model TU-50, $\tau = T/4$

the Table estimation gives an accurate estimation of the interference power caused by an unsynchronized transmitter over a selective frequency channel. This result has been validated for both systems (CP-OFDM and FBMC) in other propagation channel model (Pedestrian-A) and considering different timing offset values.

Now, let us consider the scenario depicted in Fig 5. The transmitted power of each user must guarantee a target SNR of 20 dB at the base stations. We assume that the user of interest MU_0 occupies the k^{th} cluster (18 subcarriers/cluster) and the asynchronous interferer MU_1 utilizes the two adjacent clusters $(k-1)$ and $(k+1)$. We assume that the propagation channels are stationary over the whole data frame. Also, the underlying channel models include path-loss effects, which take into account the location information of the users (Table II). Each user is perfectly synchronized with its base station.

Our aim is to evaluate the performance of the transmission between the user of interest and the reference base station BS_0. In this evaluation, we take into account the asynchronous interference caused by MU_1. The timing offset τ between the reference base station BS_0 and MU_1 is uniformly distributed

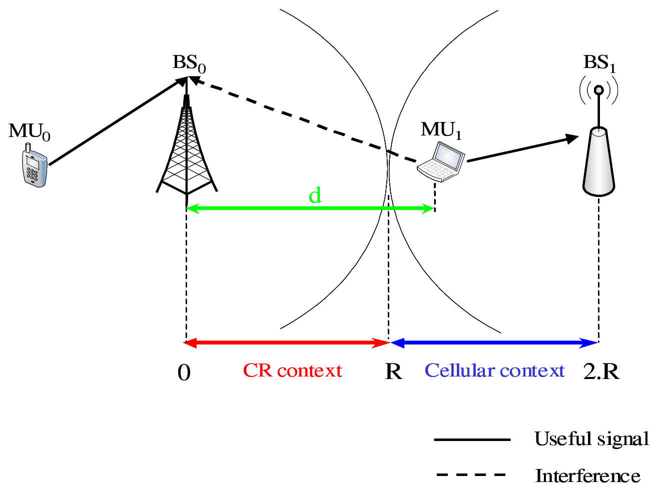

Fig. 5. Interference model: the reference user coexists with an asynchronous interferer.

on $[0, T]$.
If d varies from R and $2R$, this implies that MU_1 can move from the cell edge of cell 1 to the center of this latter. This scenario will be called a classical macro cellular context.
On the other hand, when d varies from 0 to $2R$, it implies that MU_1 can be very closed to BS_0 while transmitting to BS_1. This scenario is similar to a cognitive radio context where a secondary user (MU_1) can be very closed to the primary base station (BS_0).
This evaluation has been performed using the Monte-Carlo simulation (MC) (averaged on the transmission of 1000 frames for each channel realization), the Table estimation method (TE) using the expression (13) and also Perfect synchronized case (PS). In the PS case, it is assumed that MU_1 is synchronized with BS_0.

Fig. 6 shows the averaged capacity for each waveform versus the distance d_{MU_1,BS_0} in asynchronous and perfect synchronized case. As aforementioned, the distance between the interferer and the reference base station varies between $0.1R$ and $1.9R$. The capacity has been averaged over all random variables on which it depends : the rayleigh channel gains, the timing offsets τ and the transmitted data symbols.

In the cognitive radio case, we note a significant degradation of the capacity of the asynchronous OFDM case compared to the perfect synchronized (PS) case. On the other hand, we have a slight loss of the averaged capacity in the asynchronous FBMC compared also to the PS case. The better behavior of FBMC compared to OFDM can be explained by Fig. 1 and 2. It is clear, looking at Fig. 2, that only one adjacent subcarrier of $(k-1)^{th}$ and $(k+1)^{th}$ clusters will cause interference for the cluster of interest. Instead, looking at Fig. 1, a greater number of adjacent subcarriers will contribute to interference (more than 6 adjacent subcarriers give an interference power greater than 10^{-3} for OFDM while only one for FBMC).

In the cellular context, the interferer MU_1 is quite far from the reference BS_0 and closed to its base station BS_1. This

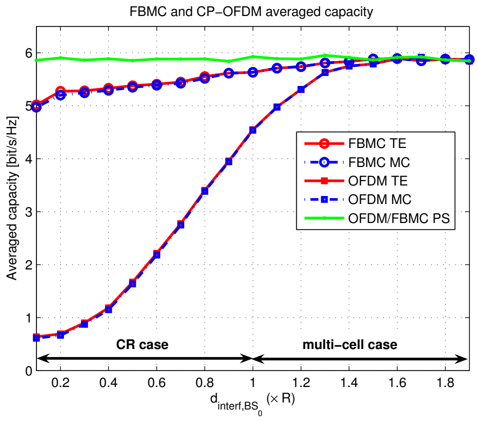

Fig. 6. FBMC and OFDM averaged capacity *vs.* distance between the interferer and the reference BS_0, $\tau \in [0, T]$

means that its transmitting power will be reduced, furthermore, the interference received by BS_0 will be more decreased by the pathloss effects. Therefore, timing synchronization errors have less significant effects on the performance in cellular networks.

In the PS case, the orthogonality between subchannels is maintained. Therefore, the capacity of the user depends only on the noise level and the rayleigh channel gain. As previously mentioned, the average SNR is equal to 20 dB which corresponds to a capacity of 6.65 $bit/s/Hz$. However, we have an averaged capacity lower than the expected 6.65 $bit/s/Hz$ in Fig. 6. This can be explained using the Jensen inequality [11]

$$log_2\left(1 + E\left[\text{SNR}\right]\right) > E\left[log_2\left(1 + \text{SNR}\right)\right]$$

It is worth noting that in this analysis, we do not consider the CP effect on the reduction of the capacity. The FBMC waveform is not concerned by this reduction due to the non-use of the cyclic prefix. Therefore, the achieved capacity gain should be more important than the obtained gain illustrated in the different results.
Moreover, comparing the table estimation method to the Monte Carlo one in Fig. 6, we find that both methods provide the same results. Hence, the results validate the interference table modeling using expression (13).

V. Conclusion

In this paper, we have analyzed the interference of CP-OFDM and FBMC systems caused by the timing synchronization errors and the multipath channel effects. We first develop a theoretical derivation of the interference in frequency selective environments. Then, we have proposed a new interference model which can be considered as a simpler alternative to the computationally exact evaluation method. We further explored an application of this model. In this application, we have shown that FBMC technique is less sensitive to the timing unsynchronization compared to the CP-OFDM due to the loss of orthogonality damaged by the non-synchronization. Moreover, the FBMC has been demonstrated as a potential alternative in cognitive radio context due to the better frequency localization of used pulse shape. Finally, we have shown, in the multi-cell context, that the timing unsynchronization has a lower significant effect on system performance for both techniques.

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